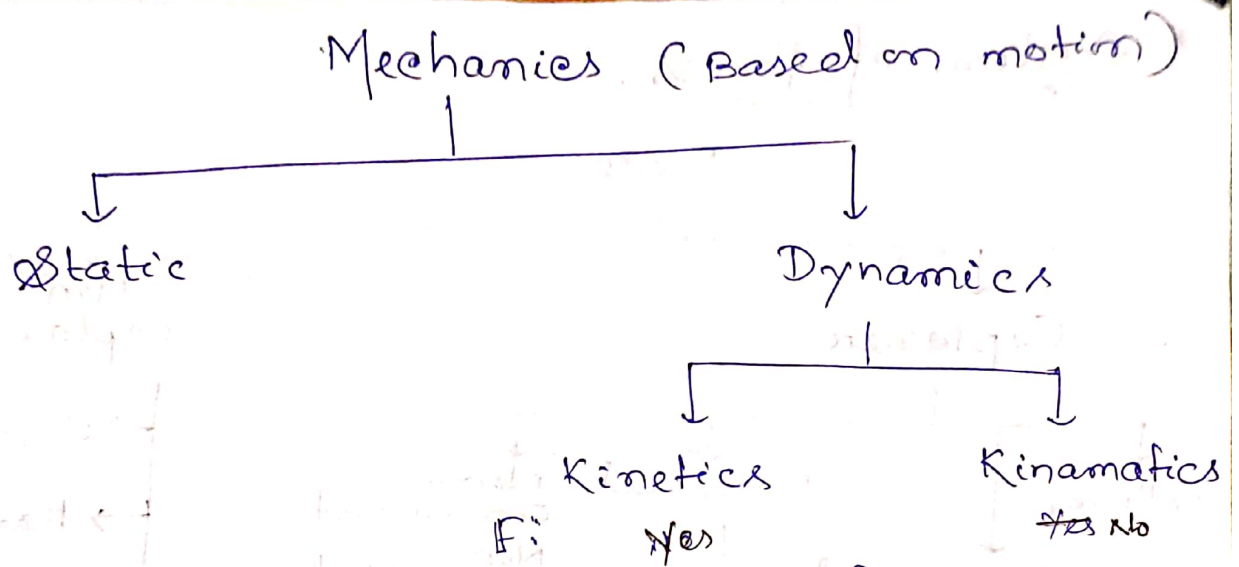




* Eng. Mech. is that branch of science which deals with the behaviour of body which is moving or in rest. Eng. Mech may be divided into 2 parts i.e. static or dynamics.

* The branch of science that deals with the study of objects which is in rest, is known as Static. On the other hand the branch of science which deals with the study of body which is moving, is known as Dynamics.

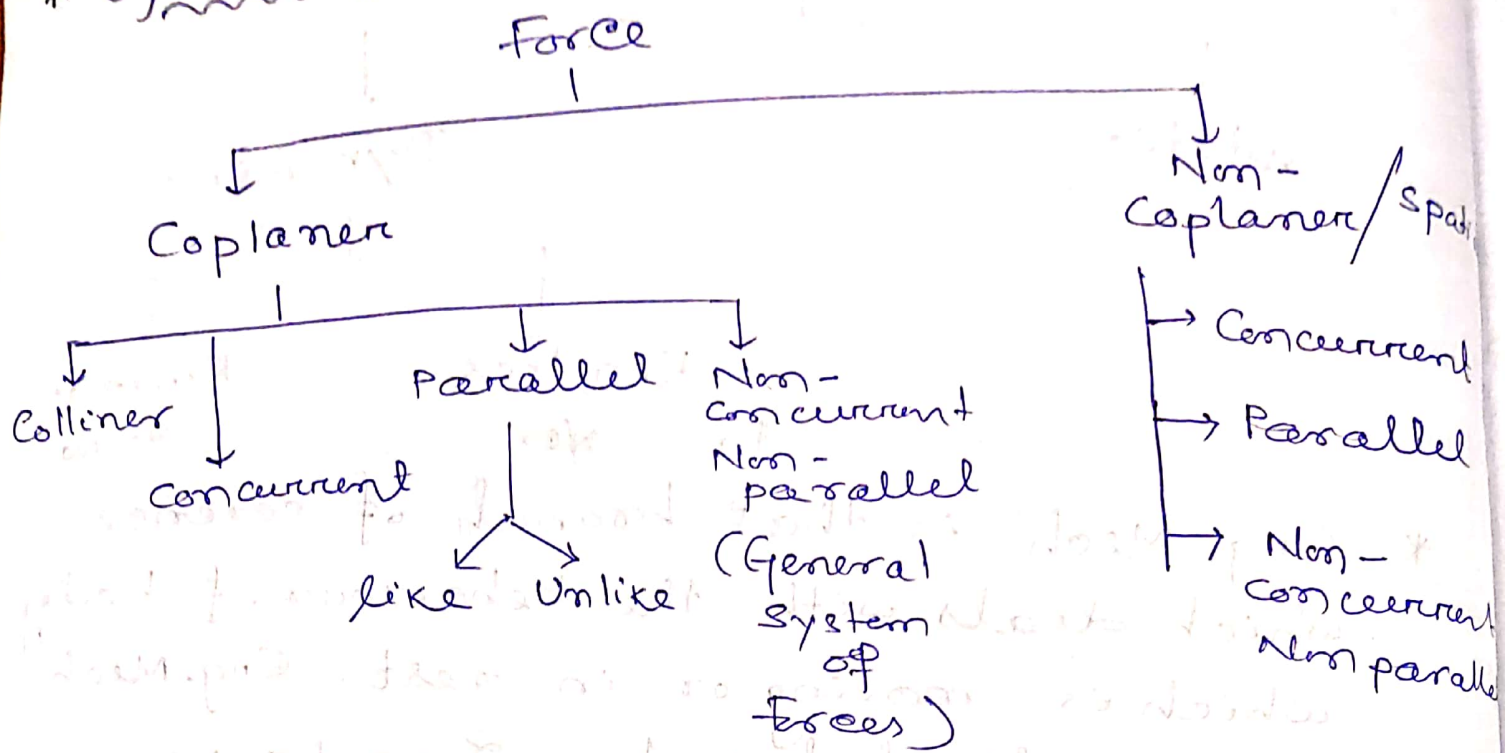
→ Dynamics is again divided into 2 parts i.e. Kinetics & Kinematics.

* The study of a body in motion, when the force which causes motion is considered, it is called Kinetics. If the cause of motion is not considered it is called Kinematics.

* Characteristics of forces :-

- | | |
|--------------|--------------------------|
| 1. Magnitude | 3. Nature (Push or Pull) |
| 2. Direction | 4. Point of application. |

* System of Forces



*. Coplanar Forces : In this system all the forces lie in a single plane

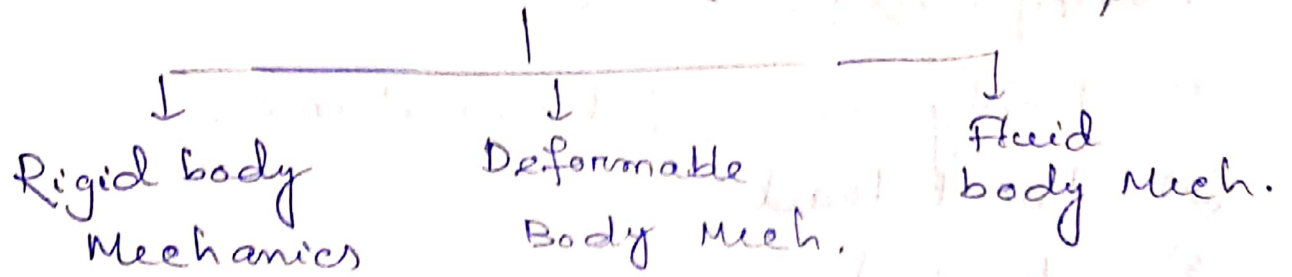
*. Non Coplanar : All forces don't lie in a single plane.

*. Coplanar - Collinear force :
In this system all the forces act along a single line & at a single plane, then that type of force known as coplanar-collinear forces

*. Coplanar - Concurrent forces :
In this system force are in same plane and all the forces meet at a common point.

*. Coplanar - parallel forces :

Mechanics (based on body)



* Non-concurrent Non parallel :

Resultant force of two forces

- (i) Resultant force of two forces
- (ii) Resultant force of two forces
- (iii) Resultant force of two forces
- (iv) Resultant force of two forces

Assumptions: -

- ① Body is rigid in nature.
- ② Point load
- ③ Particle

* Composition & Resolution of forces: - (Resultant)

1. Composition of concurrent coplanar forces:

The process of finding of a resultant force of a number of given forces is called composition of forces.

2. Type of composition of forces:

- | | | | |
|---------------------|-------|------------------------------|----------------------|
| Graphical
method | (i) | Parallelogram law of forces. | } 2 forces |
| | (ii) | Triangle " " " | |
| | (iii) | Polygon " " " | |
| | (iv) | Resolution of forces. | } More than 2 forces |

(i) Parallelogram law:

In $\triangle AEC$:

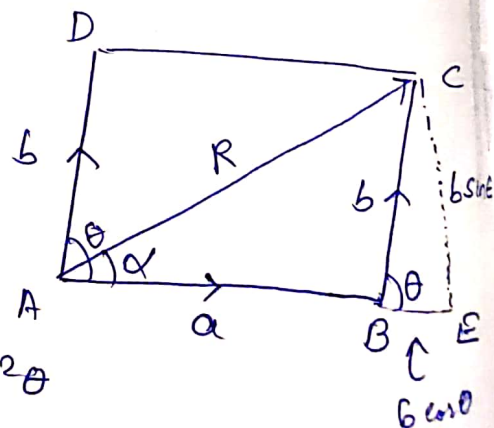
$$AC^2 = AE^2 + EC^2$$

$$= (AB + BE)^2 + (EC)^2$$

$$= (a + b \cos \theta)^2 + b^2 \sin^2 \theta$$

$$= a^2 + b^2 \cos^2 \theta + 2ab \cos \theta + b^2 \sin^2 \theta$$

$$= a^2 + b^2 + 2ab \cos \theta$$



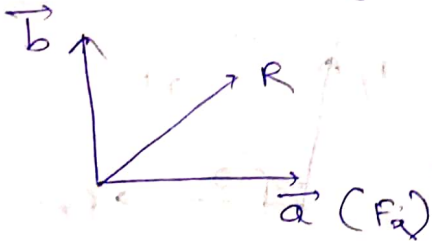
$$AC = |\vec{R}| = \sqrt{a^2 + b^2 + 2ab \cos \theta} \quad ; \theta \text{ is the angle bet}^n \vec{a} \& \vec{b}.$$

$$\tan \alpha = \frac{b \sin \theta}{a + b \cos \theta} \quad , \alpha \text{ is the angle bet}^n \vec{R} \& \vec{a}.$$

Case - 1

$$\text{If } \theta = 90^\circ, \tan \alpha = b/a$$

$$|\vec{R}| = \sqrt{b^2 + a^2}$$



Case - 2

$$\text{If } \theta = 0^\circ, |\vec{R}| = a + b$$

$$\alpha = 0^\circ$$

Case - 3

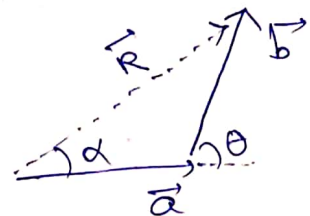
$$\text{If } \theta = 180^\circ, |\vec{R}| = a - b$$

$$\alpha = 0^\circ$$

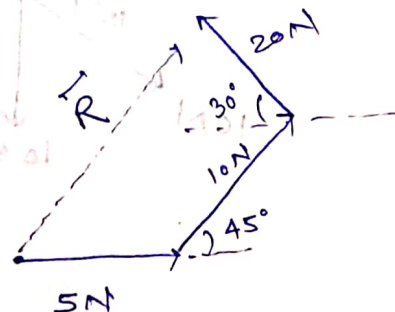
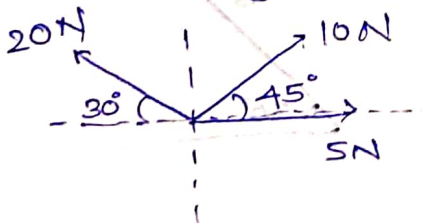
* Triangle law:

$$R = \sqrt{a^2 + b^2 + 2ab \cos \theta}$$

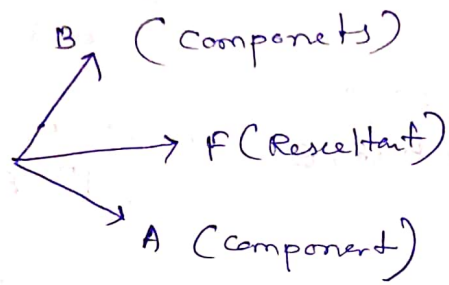
$$\alpha = \tan^{-1} \left(\frac{b \sin \theta}{a + b \cos \theta} \right)$$



* Polygon law:

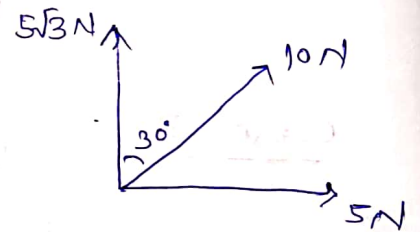
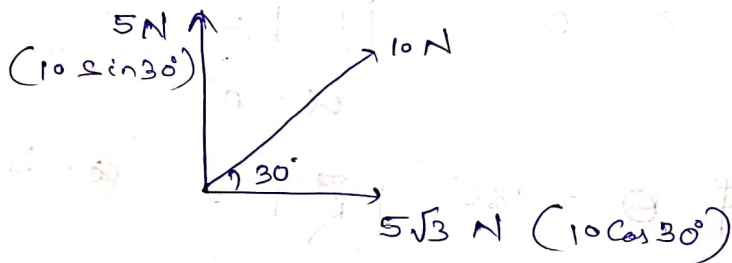
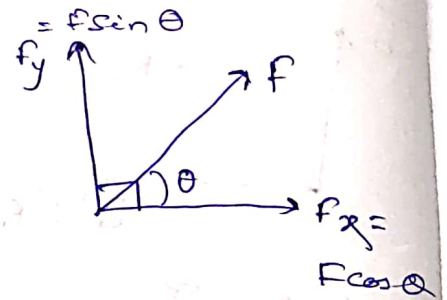


* Resolution of forces:

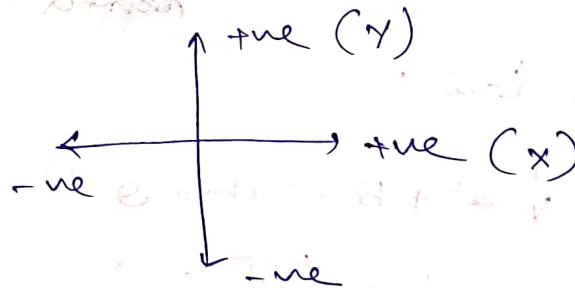


→ Special Component:

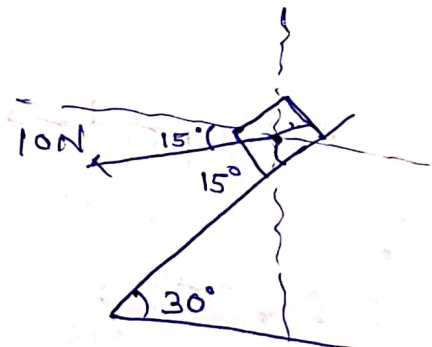
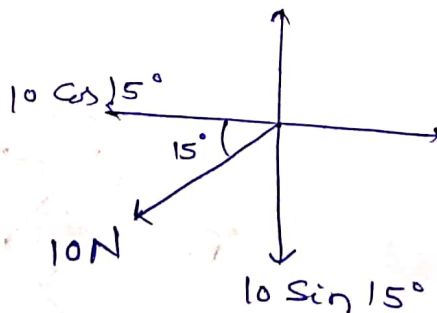
f_x & f_y are rectangular component. And angle betⁿ them is 90° .



Sign Convention:



Q.



3 special forces :

1. Weight $\rightarrow$ Always act vertically $\downarrow$
2. Normal Reaction $\rightarrow$ perpendicular to the surface.
3. Friction $\rightarrow$ Opposite to the direction of impending motion.

G:

$\rightarrow$ Resultant by using resolution method :

Step - 1 : Resolve the forces acting at an angle with x & y -axis.

- 2 : Find out summation of all horizontal forces i.e. $\sum F_x$.

- 3 : Find out summation of all vertical forces i.e. $\sum F_y$.

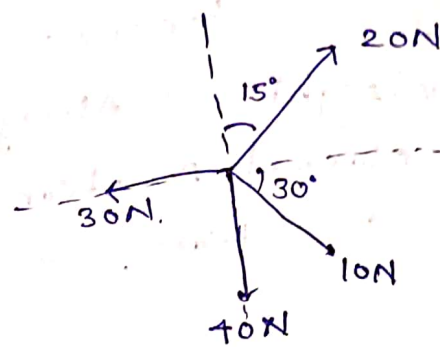
- 4 : Find the mag. of resultant i.e.

$$R = \sqrt{(\sum F_x)^2 + (\sum F_y)^2 + 2 F_x F_y \cos \theta}$$
$$= \sqrt{(\sum F_x)^2 + (\sum F_y)^2} \quad (\text{as } \theta = 90^\circ)$$

$$\tan \alpha = \frac{\sum F_y}{\sum F_x}$$

H.W

find out the resultant of all the forces.



* Free body Diagram:

This diagram is a sketch of a body which represents it as being isolated or free from its surroundings. On this sketch show only the form which ^{the} surrounding exert on ^{the} other body.

Step-1: Identify that body

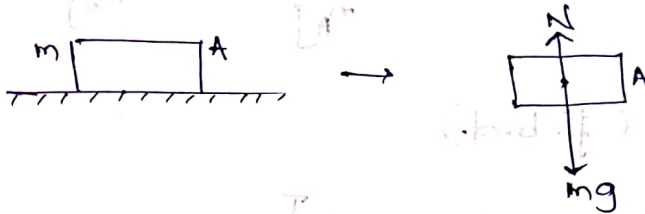
2: Isolate the body from surrounding.

3: Draw free body diagram (Consider the forces which are acting on the body)

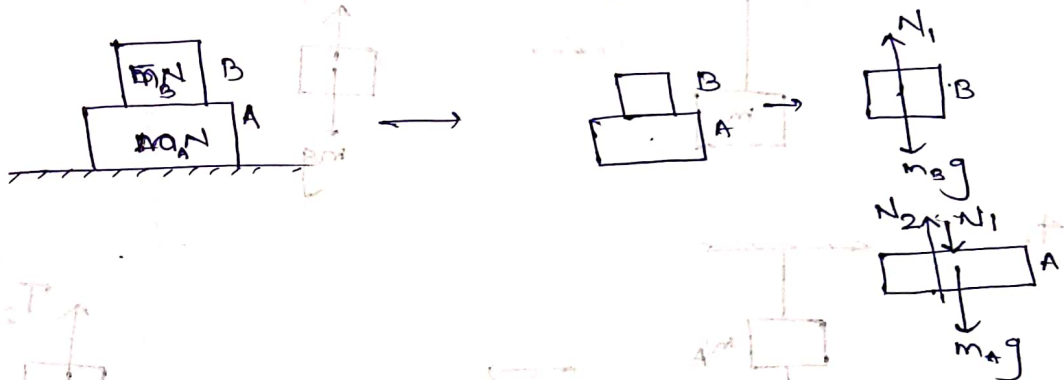
forces — $\left\{ \begin{array}{l} \text{Contact force (friction)} \\ \text{Non-Contact force (Gravitational force, Magnetic force)} \end{array} \right.$

Ex:

1.

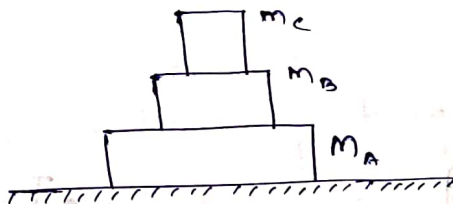


2.



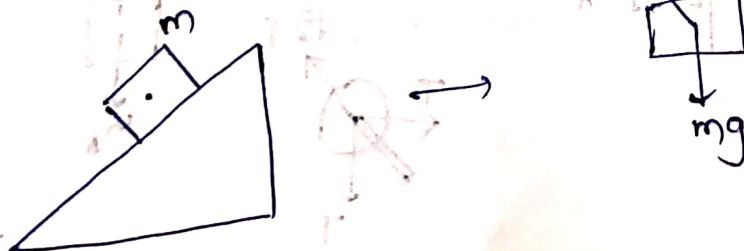
H.W

Draw f.b.d:

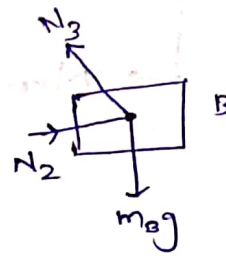
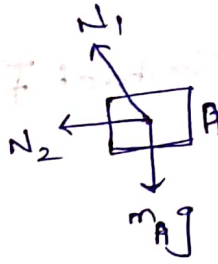
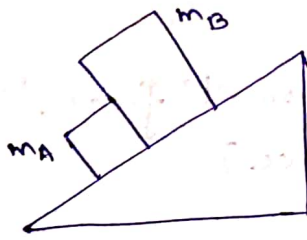


Inclined Plane (f.b.d)

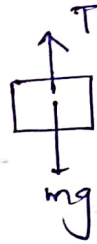
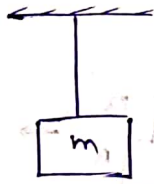
Eg: 1.



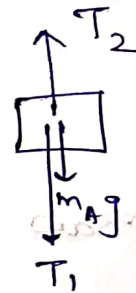
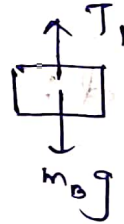
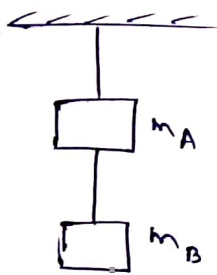
2.



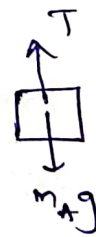
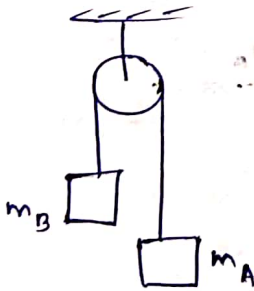
3. Wire/Rope : (f.b.d.)



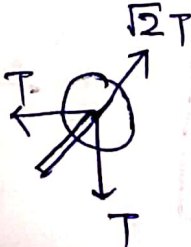
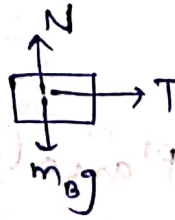
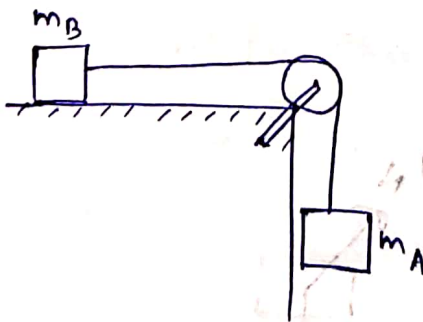
4.



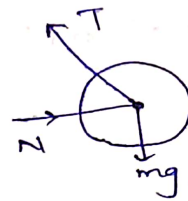
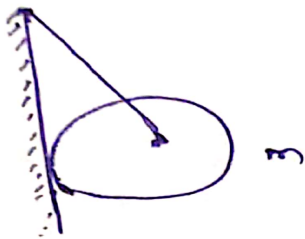
5.



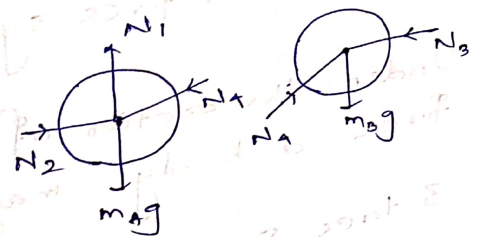
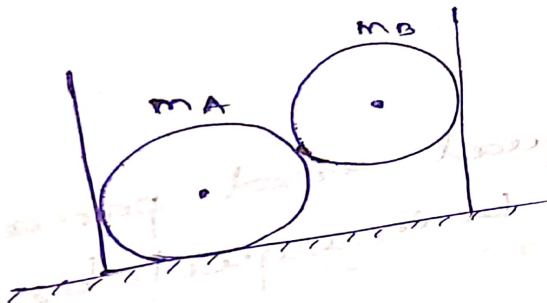
6.



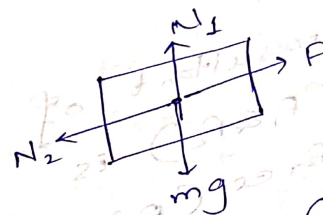
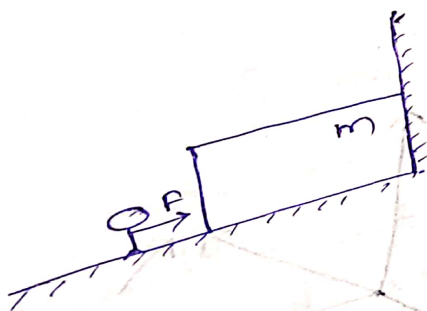
7.



8.



9.



* Conditions of Equilibrium (for concurrent & parallel forces)

Principle of equilibrium states that a stationary body which is coplaner forces (concur. & parallel) will, if the algebraic sum of moments of all the forces acting on it is zero and also

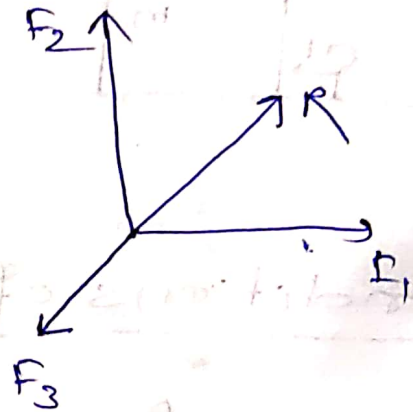
* Force law of equilibrium:

- ① 2 force system.
- ② 3 force system.
- ③ More than 3 force system.

①
→ Under the action of 2 equal and parallel force a body may not be in equilibrium.

② 3 force System:

If R is the resultant of any 2 forces (F_1 & F_2) is equal to 3rd force (F_3) then the body is in equilibrium.



③ More than 3 forces:

Lami's Theorem :-

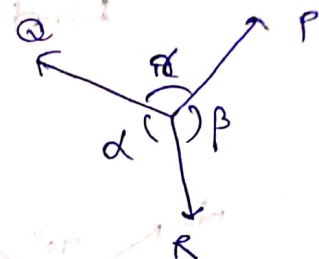
$$P \propto \sin \alpha \Rightarrow P = K \sin \alpha$$

$$Q \propto \sin \beta \Rightarrow Q = K \sin \beta$$

$$R \propto \sin \gamma \Rightarrow R = K \sin \gamma$$

$$\frac{P}{\sin \alpha} = \frac{Q}{\sin \beta} = \frac{R}{\sin \gamma}$$

(The body must be in equilibrium)



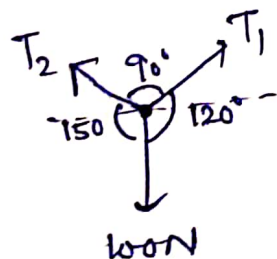
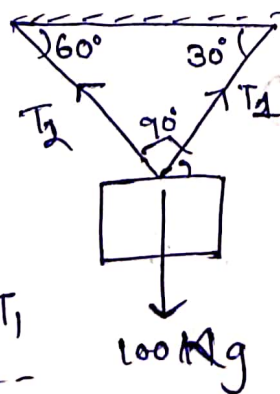
Principle of Transmissibility of forces :-

If a force at any point on a rigid body, it may also be considered to act at any other point on its line of action, provided the point is rigidly connected to the body.

This principle is only applicable for rigid body.

By applying this principle only the state of a body remains same, but the internal stress will change.

G:



Find T_1 & T_2 ?

Applying Lami's theorem.

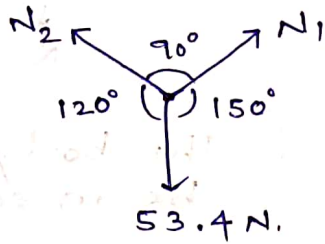
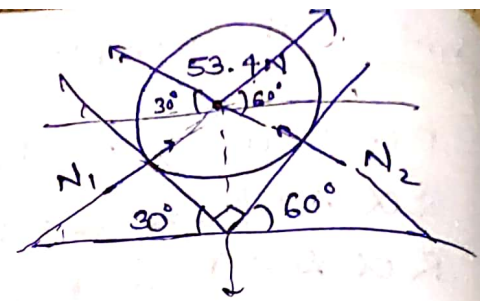
$$\frac{1000}{\sin 90^\circ} = \frac{T_2}{\sin 120^\circ} = \frac{T_1}{\sin 150^\circ}$$

$$\Rightarrow T_1 = 1000 \times \frac{1}{2} = 500 \text{ N}$$

$$T_2 = 1000 \times \frac{\sqrt{3}}{2} = 500\sqrt{3} \text{ N}$$

Ex:

find out N_1 & N_2 ?



According to Lami's law

$$\frac{N_1}{\sin 120^\circ} = \frac{N_2}{\sin 150^\circ} = \frac{53.4}{\sin 90^\circ}$$

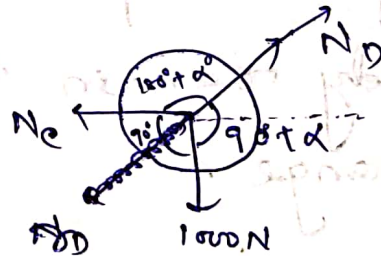
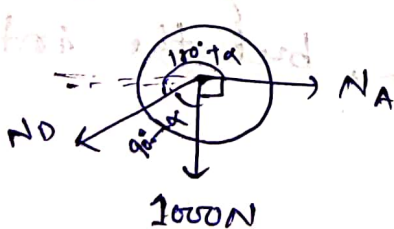
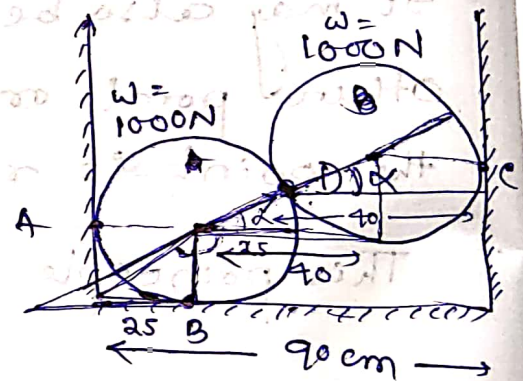
$$N_1 = 53.4 \times \frac{\sqrt{3}}{2} = 26.7\sqrt{3} \text{ N}$$

$$N_2 = 53.4 \times \frac{1}{2} = 26.7 \text{ N}$$

Ex:

2 spheres each of wt. 1000 N & radius 25 cm, rest in a horizontal channel.

find reaction at the point of contact at A, B, C, D.



$$50 \cos \alpha = 40$$

$$\alpha = \cos^{-1}\left(\frac{4}{5}\right)$$

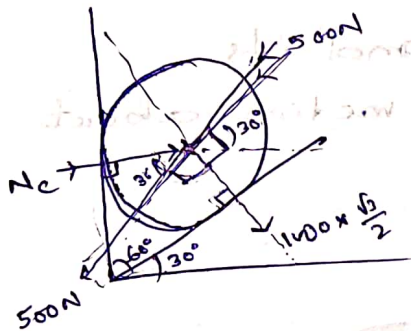
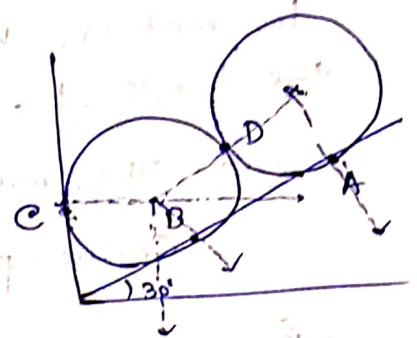
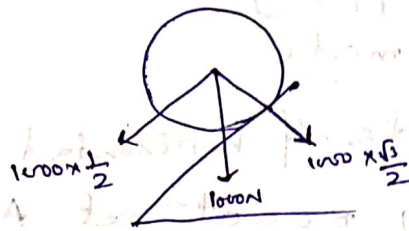
$$= 37^\circ$$

$$\frac{1000}{\sin(180^\circ + \alpha)} = \frac{N_A}{\sin(90^\circ - \alpha)} = \frac{N_D}{\sin(90^\circ)}$$

$$N_D = \frac{1000}{\sin(180^\circ + \alpha)} \times \frac{\sin(90^\circ)}{\sin(90^\circ - \alpha)} = 1000 \times \frac{1}{\cos \alpha} = 1250 \text{ N}$$

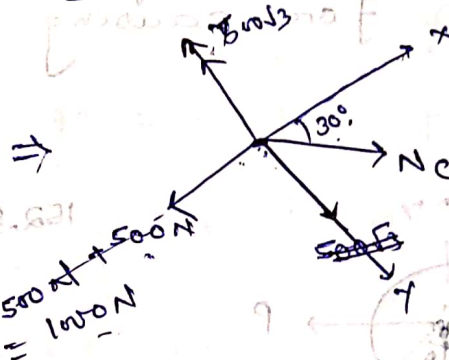
4.10 2 identical rollers each of wt. 1000 N are supported by an inclined plane & a vertical wall. find reaction at A, B, C, & D.

Soln:



$$N_A = 500\sqrt{3} \text{ N}$$

$$N_D = 500 \text{ N}$$



$$N_c \cos 30^\circ = 1000$$

$$\Rightarrow N_c = \frac{1000}{\cos 30^\circ}$$

$$N_c \sin 30^\circ = 500\sqrt{3}$$

$$N_c = 2000$$

$$N_c \times \frac{\sqrt{3}}{2} = 500$$

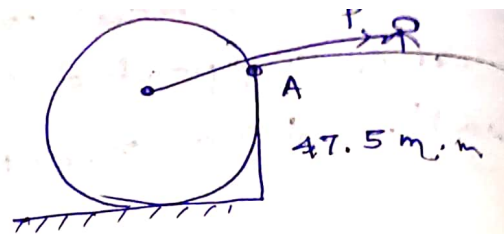
$$N_c = \left(\frac{1000}{\cos 30^\circ} \right) \sin 30^\circ = 1000$$

$$N_c \times \frac{\sqrt{3}}{2} = 500$$

$$N_c = \left(\frac{1000}{\cos 30^\circ} \right) \sin 30^\circ = 1000$$

$$N_c = 1000$$

Q. Diameter = 400 mm
 $W = 400 \text{ N}$
 Is to be subjected to motion, while applying the force P . Determine

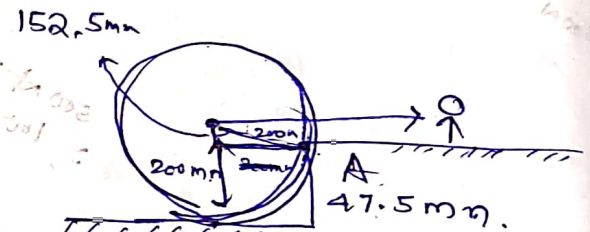
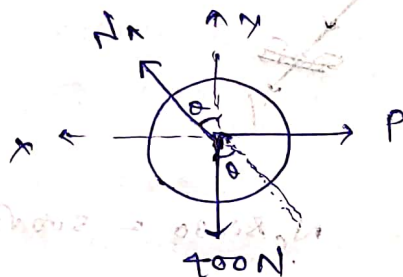


① The magnitude of horizontal force P , and corresponding reaction at A .

② The mag. of min. force P and its direction for causing the motion about point A .

Solⁿ:

①



$$\cos \theta = \frac{152.5}{200} = \frac{61}{80}$$

$$\theta = \cos^{-1} \left(\frac{61}{80} \right) = 40.31^\circ$$

$$N_A \cos(40.3) = 400$$

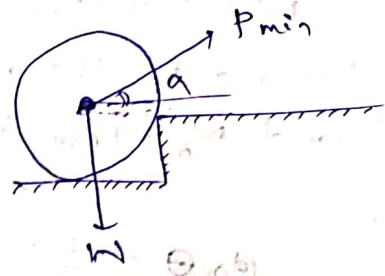
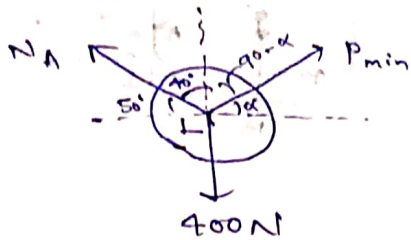
$$\Rightarrow N_A = \frac{400}{\cos(40.3)} = 524.47 \text{ N}$$

$$P = N_A \sin \theta$$

$$= 524.47 \times \sin(40.3)$$

$$= 339.22 \text{ N}$$

11



$$\frac{P_{\min}}{\sin(130^\circ)} = \frac{N_A}{\sin(90^\circ + \alpha)} = \frac{400}{\sin(130^\circ - \alpha)}$$

OR

$$\sum F_x = 0$$

$$P \cos \alpha - N_A \sin 40^\circ = 0$$

$$N_A = \frac{P \cos \alpha}{\sin 40^\circ} \quad \text{--- (I)}$$

$$\Rightarrow P \cos \alpha = N_A \sin 40^\circ$$

$$\Rightarrow \text{--- (II) / (I) ---}$$

from (I) & (II)

$$\sum F_y = 0$$

$$P \sin \alpha + N_A \cos 40^\circ = 400$$

$$\Rightarrow P \sin \alpha = 400 - N_A \cos 40^\circ$$

$$\text{--- (II)}$$

$$P_{\min} =$$

$$\frac{400}{\sin \alpha + \cos \alpha \cdot \cot 40^\circ}$$

$$\frac{d}{d\alpha} [\sin \alpha + \cos \alpha \cdot \cot 40^\circ] = 0$$

$$\Rightarrow \cos \alpha - \cot 40^\circ \sin \alpha = 0$$

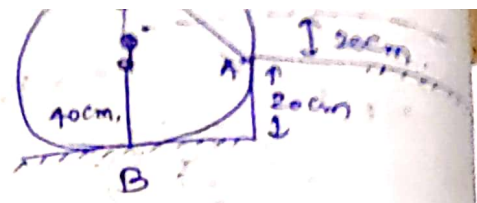
$$\Rightarrow \cot \alpha = \cot 40^\circ$$

$$\Rightarrow \alpha = 40^\circ$$

$$P_{\min} = \frac{400}{\sin 40^\circ + \cos 40^\circ \cdot \cot 40^\circ}$$

$$= 257.11 \text{ N.}$$

Q.2. $W = 3000 \text{ N}$.
Find out the mag. of
 P and N_A , $P_{\min}$?



Solⁿ:

$$\cos \theta = \frac{20}{40} = \frac{1}{2}$$

$$\theta = 60^\circ$$

$$\text{So } \alpha = 30^\circ$$

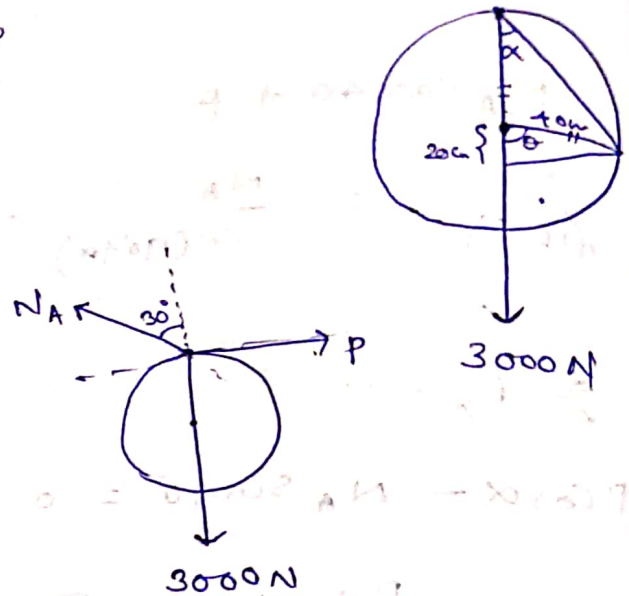
$$N_A \cos 30^\circ = 3000 \text{ N}$$

$$N_A = 2000\sqrt{3} \text{ N}$$

$$N_A \sin 30^\circ = P$$

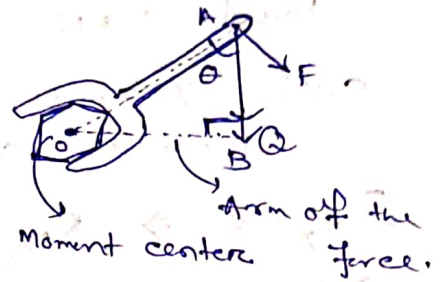
$$\Rightarrow P = 2000\sqrt{3} \times \frac{1}{2}$$

$$= 1000\sqrt{3} \text{ N}$$



Method of Moment: (Moment of force)

$$\text{Moment} = (F \times OA)$$



2 Mark

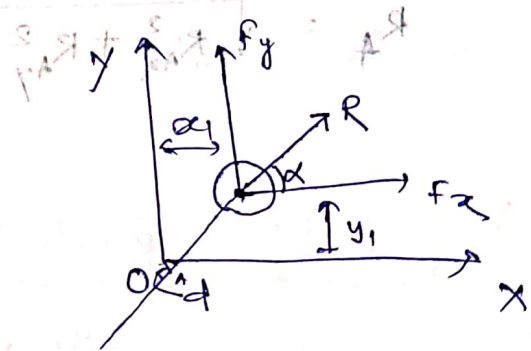
Theorem of Varignon:

It states that the sum of moments of all the forces will be equal to the moment produced by the resultant of the forces.

$$M_o = F_x y_1 - F_y x_1$$

$$= R \times OA$$

$$= R \times d$$



* Varignon's theorem can be used to find out the location of resultant for coplanar non-concurrent forces.

Eg: Find the location of resultant force for the given sys. of forces.

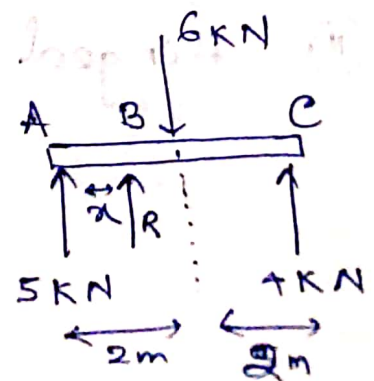
Soln: $\sum F_y = 0$ (for equilib.)

$$\Rightarrow 5 + 4 - 6 = 3 \quad (\text{so not in equilibrium})$$

$$R = 3 \text{ kN}$$

$$-R \cdot x = 5 \times 0 + 6 \times 2 - 4 \times 4$$

$$-3x = -4 \Rightarrow x = 4/3 = 1.33 \text{ m}$$



Ex: A crane is pivoted at end B, and is supported by a rod at point A. Determine the reaction at point A & B with the vertical load 5 kN at C.

Soln: System is, non-concurrent, non-parallel, coplanar

$$\sum F_x = 0 \Rightarrow R_{Ax} = R_B \quad \text{--- (1)}$$

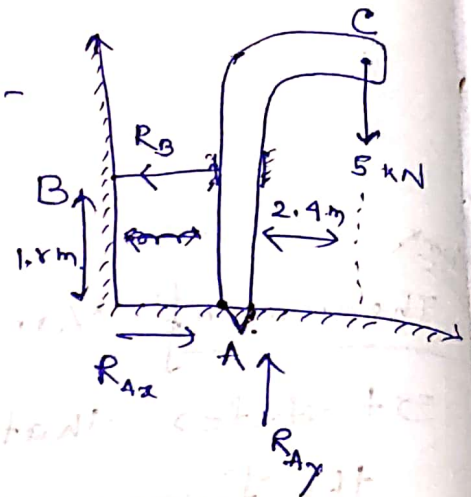
$$\sum F_y = 0 \Rightarrow R_{Ay} = 5 \text{ kN.} \quad \text{--- (2)}$$

$$\sum M_o = 0$$

$$\Rightarrow 5 \times 2.4 - R_B \times 1.8 = 0$$

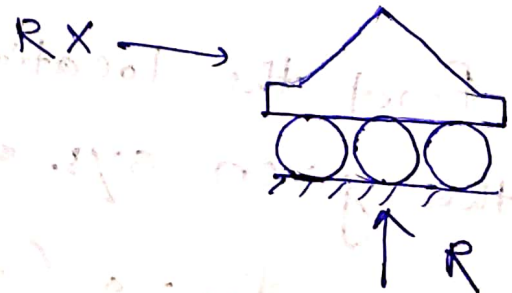
$$\Rightarrow R_B = \frac{5 \times 2.4}{1.8} = \frac{20}{3} = 6.66 \text{ kN.}$$

$$R_A = \sqrt{R_{Ax}^2 + R_{Ay}^2} = \sqrt{(5)^2 + (6.66)^2} = 8.32 \text{ kN.}$$

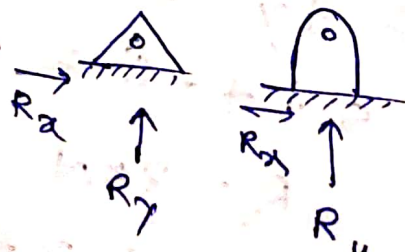


* Reaction at Supports:

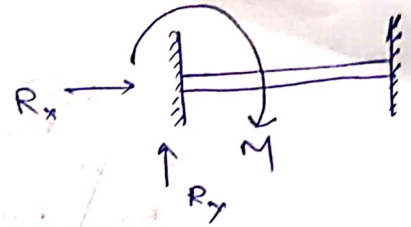
① Roller Support:



② Hinged Support:

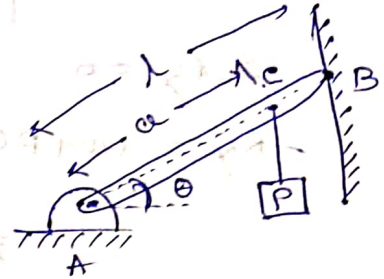
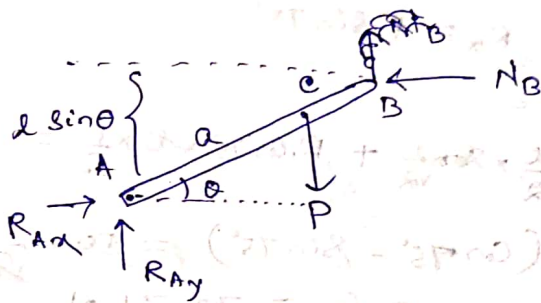


III) Fixed Support:



Ex. find R_B ?

Soln:



$$\sum F_y = 0$$

$$\sum F_x = 0$$

$$\sum M_o = 0$$

$$\Rightarrow R_{Ay} = P$$

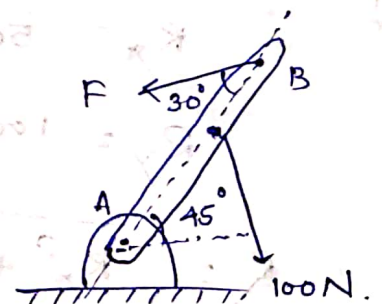
$$\Rightarrow R_{Ax} = N_B$$

$$\Rightarrow P \times a \cos \theta = N_B \cdot l \sin \theta$$

$$\Rightarrow N_B = \frac{P \cdot a}{l} \cot \theta$$

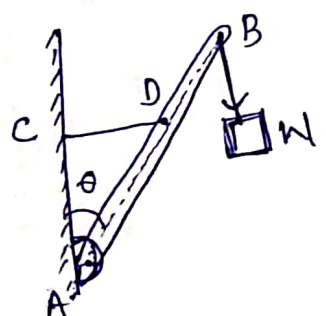
Ass. 3.1
H.W.

A bar AB of wt. 100N is hinged at point A and pulled by a cable attached at point B by a force. Find F and the mag. & direction of the reaction of resultant at point A?

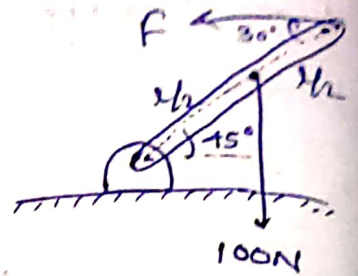
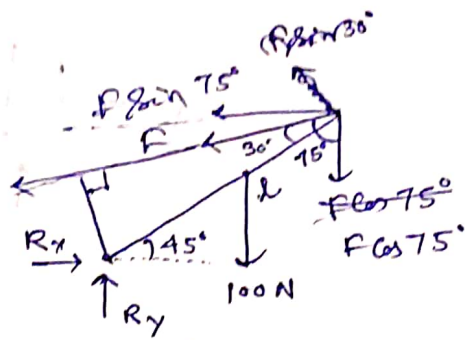


Ass. 3.2

Find the tensile force in the string & neglect the wt. of the bar?



31



$$\sum F_y = 0 \quad \sum F_x = 0$$

$$R_y = 100 + F \cos 75^\circ, \quad R_x = F \sin 75^\circ$$

$$\sum M_o = 0$$

$$\Rightarrow 100 \times \frac{l}{2} \times \sin 45^\circ + F \cos 75^\circ \times l \times \frac{1}{\sqrt{2}} = F \sin 75^\circ \times l \times \frac{1}{\sqrt{2}}$$

$$\Rightarrow -\frac{F}{\sqrt{2}} (\cos 45^\circ - \sin 75^\circ) = 50 \times \frac{1}{\sqrt{2}}$$

$$\Rightarrow F = 50\sqrt{2} \approx 70.71 \text{ N}$$

3.2.

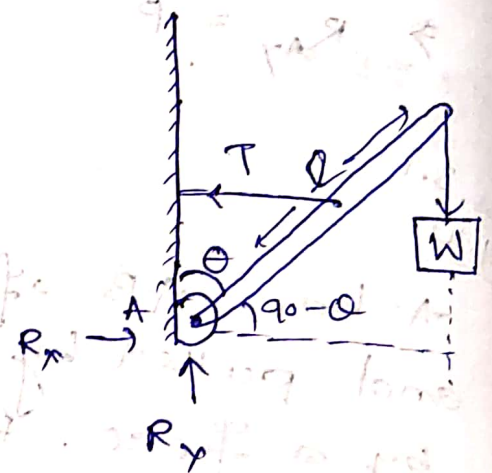
$$\sum F_x = 0 \quad \sum F_y = 0$$

$$W = R_y, \quad R_x = T$$

$$\sum M_o = 0$$

$$\Rightarrow W \times l \times \sin \theta = T \frac{l}{2} \cos \theta$$

$$\Rightarrow T = W \tan \theta$$



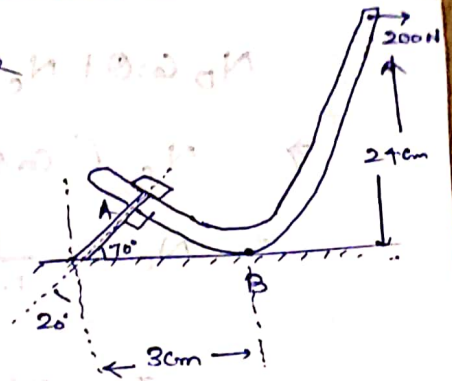
3.1

$$R_x = 50\sqrt{2} \times \sin 75^\circ \approx 68.3 \text{ N}$$

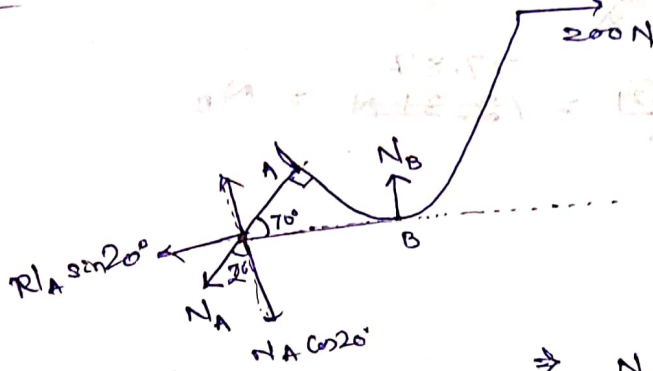
$$R_y = 100 + 50\sqrt{2} \times \cos 75^\circ \approx 118.3 \text{ N}$$

$$R = \sqrt{R_x^2 + R_y^2} = 136.6 \text{ N}$$

Q. Determine the mag. of the pull exerted on the nail, if the horizat. force of 200 N is applied to handle of the nail puller.



Solⁿ:



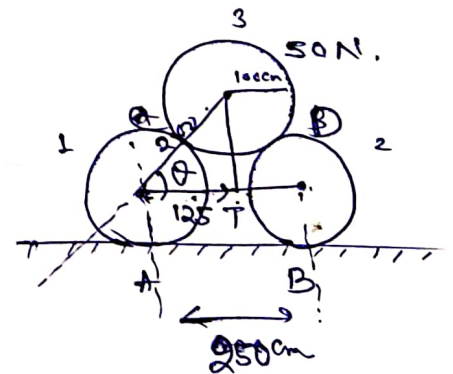
$$\sum M_B = 0$$

$$\Rightarrow N_B \times 0 + N_A \sin 20^\circ \times 0 + 200 \times 24 = N_A \cos 20^\circ \times 3$$

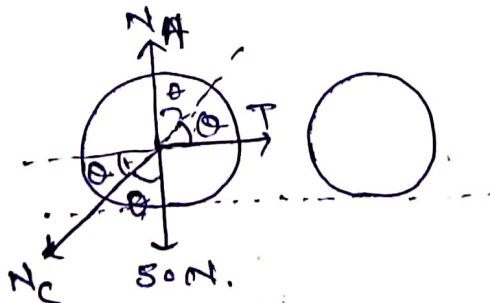
$$\Rightarrow N_A = \frac{200 \times 24}{\cos 20^\circ \times 3}$$

$$= 1702.68 \text{ N.}$$

Q. 3 identical balls are arranged as shown in figure. Find out N_C , N_D , N_A and N_B and tension in the string.



Solⁿ:

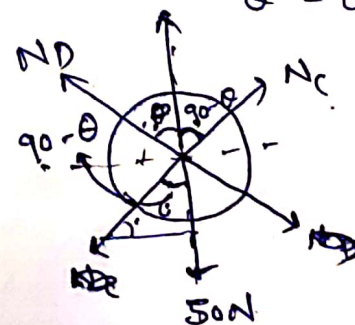


$$\cos \theta = \frac{125}{250} = \frac{5}{8}$$

$$\theta = \cos^{-1} \left(\frac{5}{8} \right) = 51.31^\circ$$

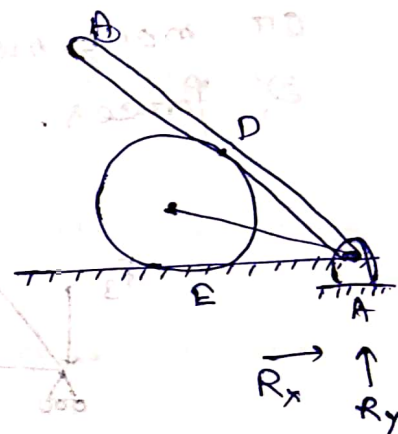
$$N_C \cos \theta + 50 = N_A$$

$$N_C \sin \theta = T$$



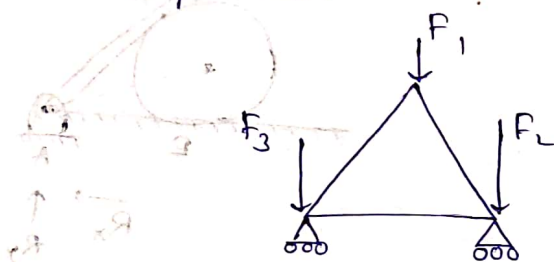
Q. A smooth cylinder of radius 10 cm, resting on a horizontal surface supports a bar AB of length 30 cm, which is hinged at A. The wt. of bar is 50 N. The cylinder is kept from rolling away by a string AO, of length 20 cm. Assume all surfaces to be smooth find the tension to the string.

Soln:

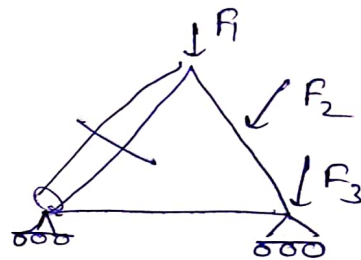


TRUSS: It is a system of uniform bars or members (of circular, angle, channel section) joint together at their ends by welding or riveting and constructed to supports loads.

Frame: It is a structure of several bars or members pinned together and one or more members subjected to more than 2 forces.



Truss



Frame

1. In truss force act at the end points of members while in case of frame it may act at the joints as well as in the middle of joints or anywhere betn the joints.
2. In a ~~frame~~ ^{truss} the forces are only axial force i.e. tensile act on the member.

In a frame along with axial force bending moment or shear force may act on the member.

Plane Truss: If forces acting on each member lies in a single plane then the truss is known as plane truss.

Space Truss: If forces don't lie in a single plane then it is called space Truss.

Plane frame:

Space frame:

* Characteristics of Truss / Assumption:

1. Members are considered as massless (unless otherwise mention)
2. Loads on the truss are applied at the joint only.
3. Members are in tension, compression or zero force.
4. The joints of a simple truss are assumed to be in connection and frictionless.
5. The truss is statically determinate.
6. The members are straight and rigid.
- 7.

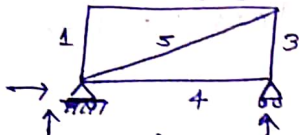
* Determination of axial forces:

1. Method of joints. ($\sum F_x = 0, \sum F_y = 0$)
2. Method of section. ($\sum F_x = 0, \sum F_y = 0, \sum M_p = 0$)
3. Graphical Method. (Not in syllabus)

* Types of Truss Frame :

Perfect / Determinate
Frame Truss

$$(M = 2j - r \text{ or } 3)$$



m = no. of member of truss

j : no. of joint

r : no. of reaction.

Collapsible /
Deficient /
Unstable

$$(m < 2j - 3)$$

Imperfect
($m \neq 2j - 3$)

Over Rigid /
Redundant
or
Indeterminate

$$(m > 2j - 3)$$

(Stable)

Determinacy :

→ Statistically determinate :

2D Truss:

$$m = 2j - 3$$

→ $m + 3 - 2j = 0$ → Stable & Determinate

$m + 3 - 2j > 0$ → Stable & Undeterminate

$m + 3 - 2j < 0$ → Unstable

3D Truss

$$m = 3j - 6$$

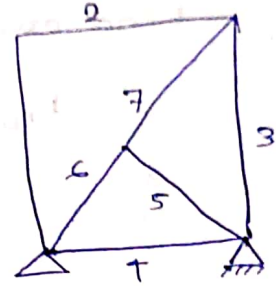
→ $m + 6 - 3j = 0$ Stable & Determinate

$m + 6 - 3j > 0$ Stable & Undeterminate

$m + 6 - 3j < 0$ Unstable.

Q. $m = 7$
 $j = 5$
 $r = 3$

$7 + 3 - 10 = 0$ (stable & determinate)

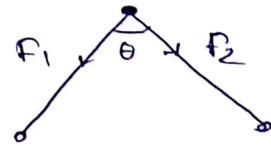


Tricks to solve

(1) Principle of zero force:

Case - 1

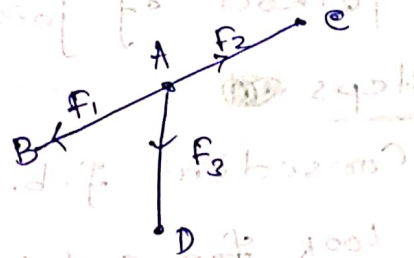
$\theta \neq 180^\circ$
 $F_1 = F_2 = 0$



Case - 2

$\text{If } F_1 = F_2 \text{ then}$

$F_3 = 0$



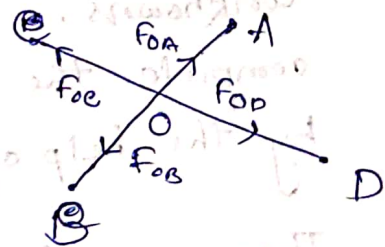
Case - 3

$\text{If } OA \text{ \& } OC \text{ are}$

collinear and OC and OD
are collinear and no

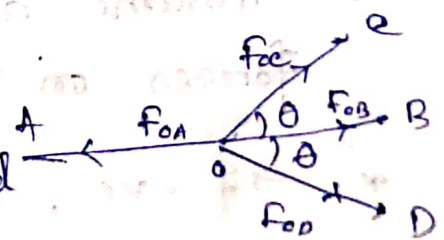
force act on point 'O' then

$F_{OA} = F_{OB}$ & $F_{OC} = F_{OD}$



Case - 4

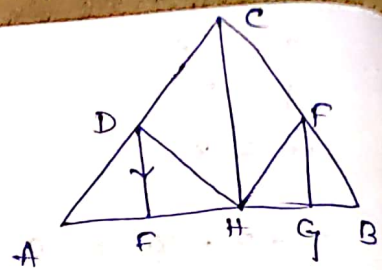
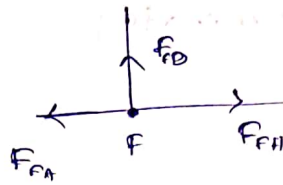
Here if OA & OB are collinear
and OC & OD are equally inclined
to OB then $F_{OC} = F_{OD}$ & $F_{OA} \neq F_{OB}$



Ex: Find out the force F_{FB} ?

①

$$F_{FD} = 0 \quad (\text{Ans-2})$$

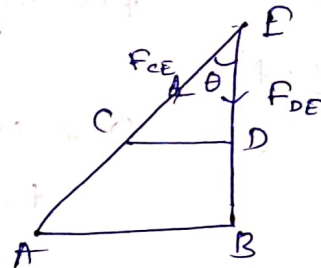


②

$$F_{CD} \text{ \& } F_{DE} = ?$$

(Ans-1)

$$F_{CD} = F_{DE} = 0$$



① Method of joints:

Steps

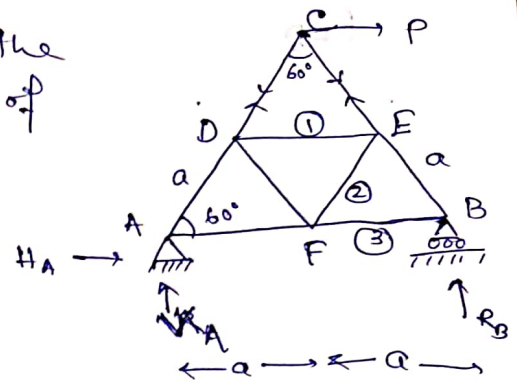
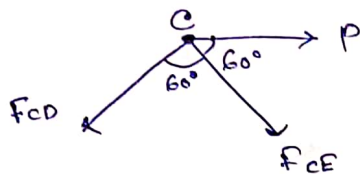
- * Consider f.b. diagram of entire truss.
- * Look for a joint at which there are only 2 unknowns. If such a joint is not found compute the reaction forces (support reaction) by the help of movement of a force.
- * Then consider a joint that has 2 links attached where one force acting on the joint is known.
- * Assume and make the direction of axial forces on the diagram.
- * If -ve, assumed direction simply has to be reversed.
- * Repeat this process until all the forces are calculated.

* Draw a table showing all the forces acting on all the members.

eg. find the axial forces in the members 1, 2, 3 by method of joints.

Solⁿ:

Consider 'C'



According to Lami's theorem,

$$\frac{P}{\sin 60^\circ} = \frac{F_{CD}}{\sin 60^\circ} = \frac{F_{CE}}{\sin 120^\circ}$$

$\Rightarrow F_{CD} = P$ & $F_{CE} = P$: So CE in compression.

$$\sum M_A = 0$$

$$\Rightarrow -R_B \times 2a + P \times \sqrt{3}a = 0$$

$$\Rightarrow R_B = \frac{\sqrt{3}P}{2}$$

$$\sum F_x = 0$$

$$\Rightarrow H_A = -P$$

$$\sum F_y = 0$$

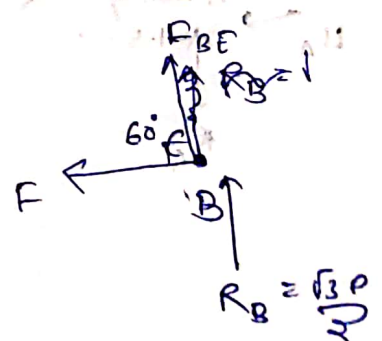
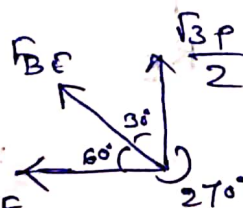
$$\Rightarrow R_B + V_A = 0$$

$$\Rightarrow V_A = -\frac{\sqrt{3}P}{2}$$

Consider 'B'

$$\frac{F_{BE}}{\sin 120^\circ} = \frac{F_{BF}}{\sin 30^\circ} = \frac{R_B/2}{\sin 60^\circ}$$

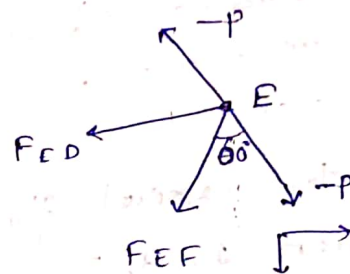
$$F_{BF} = P/2 = F_3 \quad F_{BE} = -P$$



Consider: E

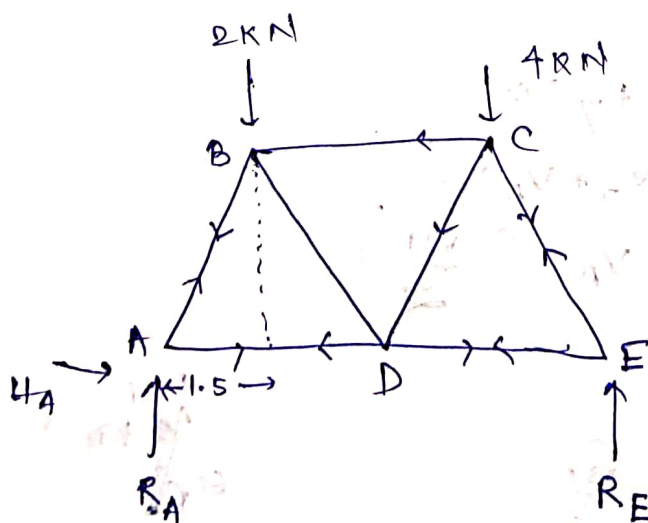
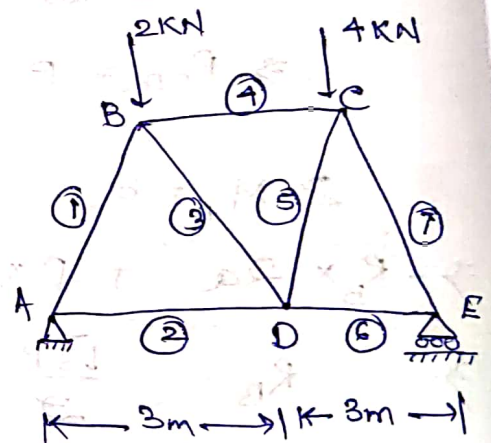
$$\sum f_x = 0$$

F_{ED}



Egr 2

Find out the magnitude and nature of forces in all the members.



$$\sum M_A = 0$$

$$\Rightarrow (2 \times 1.5) + (4 \times 4.5) - (R_E \times 6) = 0$$

$$\Rightarrow R_E = \frac{3 + 18}{6} = \frac{21}{6} = 3.5 \text{ N}$$

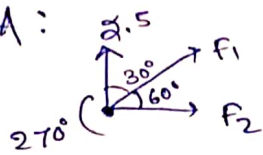
$$\sum f_x = 0$$

$$\Rightarrow H_A = 0$$

$$\sum f_y = 0 \Rightarrow R_A + R_E = 6$$

$$\Rightarrow R_A = 6 - 3.5 = 2.5$$

Joint A:

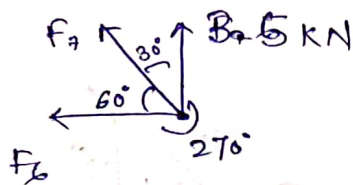


$$\frac{F_1}{\sin 90^\circ} = \frac{F_2}{\sin 30^\circ} = \frac{2.5}{\sin 60^\circ}$$

$$F_1 = \frac{2.5 \times 2 \times (-1)}{\sqrt{3}} = -5/\sqrt{3} = -2.88 \text{ KN}$$

$$F_2 = 1.44 \text{ KN}$$

Joint E

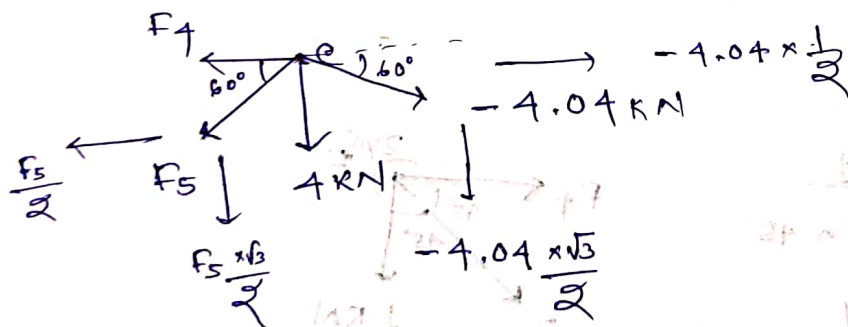


$$\frac{F_7}{\sin 270^\circ} = \frac{F_6}{\sin 30^\circ} = \frac{3.5}{\sin 60^\circ}$$

$$\Rightarrow F_7 = \frac{3.5 \times 2}{\sqrt{3}} (-1) = \frac{7}{\sqrt{3}} = -4.04 \text{ KN [C]}$$

$$F_6 = 2.02 \text{ KN [C]}$$

Joint C



$$F_4 + \frac{F_5}{2} = -2.02$$

$$2F_4 + F_5 = -4.04 \quad \text{--- (1)}$$

$$F_5 \times \frac{\sqrt{3}}{2} = 4 - 3.498 = 0.502$$

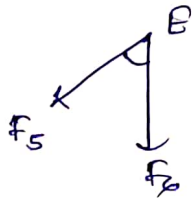
$$F_5 = \frac{0.502 \times 2}{\sqrt{3}} = 0.57 \text{ KN}$$

$$2F_4 = -4.04 + 0.57 = -3.47$$

$$F_4 = -1.735$$

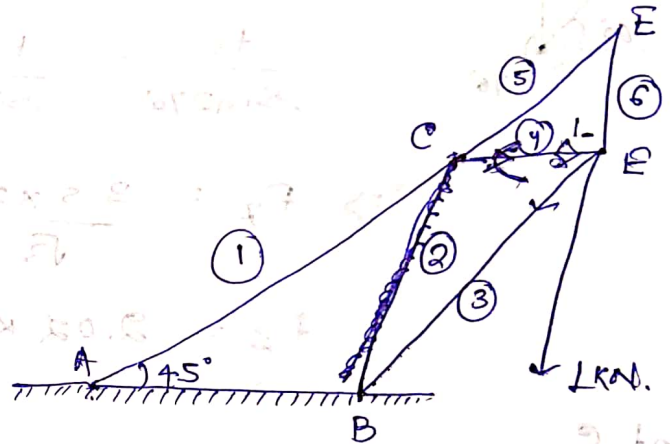
Eg. 3

Joint E:



$$F_5 = F_6 = 0$$

(Can't)



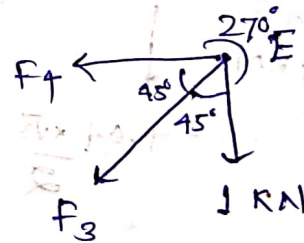
Joint D:

$$\frac{F_4}{\sin 45^\circ} = \frac{F_3}{\sin 270^\circ} = \frac{1}{\sin 45^\circ}$$

$$F_4 = 1 \text{ kN [T]}$$

$$F_3 = -\sqrt{2} \text{ kN}$$

$$= -1.414 \text{ kN} = -0.707 \text{ kN. [c]}$$

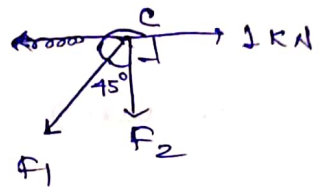


Member	Mag. of force	Nature
1	1.414 kN	T
2	-1 kN	c
3	-0.707 kN	c
4	1	T
5, 6	0	

$$F_2 = -\frac{F_1}{\sqrt{2}} = -1 \text{ kN}$$

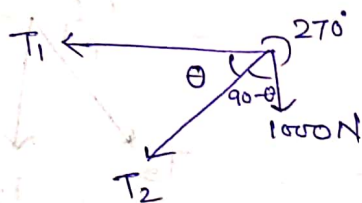
$$\frac{F_1}{\sqrt{2}} = 1 \text{ kN}$$

$$F_1 = \sqrt{2} \text{ kN}$$



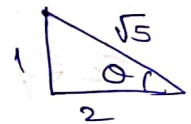
Ans.
Q.4.

At Point E



$$\frac{T_1}{\sin(90-\theta)} = \frac{T_2}{\sin 270^\circ} = \frac{1000}{\sin \theta}$$

$$\tan \theta = \frac{1}{2}$$



$$T_1 = \frac{1000}{\sin \theta} \times \cos \theta = \frac{1000}{\tan \theta} = 2000 \text{ N}$$

$$T_2 = \frac{1000}{1/\sqrt{5}} (-1) = -1000\sqrt{5} \text{ N (Compression)} \\ \approx -2236 \text{ N}$$

At point C

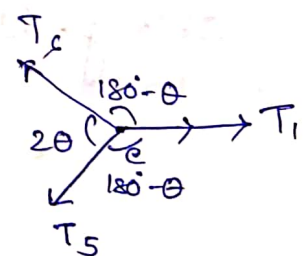
$$\frac{\sin T_1}{\sin 2\theta} = \frac{T_4}{\sin(180-\theta)} = \frac{T_5}{\sin(180-\theta)}$$

$$\Rightarrow T_4 = \frac{T_1 \times \sin \theta}{\sin 2\theta}$$

$$= \frac{2000 \times \frac{1}{\sqrt{5}}}{\frac{4}{5}}$$

$$= 500\sqrt{5} \approx 1118 \text{ N}$$

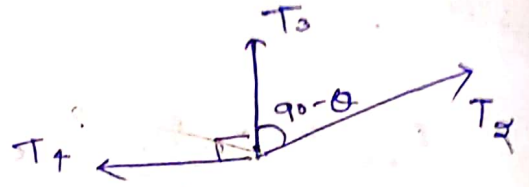
$$T_5 = \frac{T_1 \times \sin \theta}{\sin 2\theta} \approx 1118 \text{ N}$$



$$\sin 2\theta = \frac{2 \tan \theta}{1 + \tan^2 \theta} \\ = \frac{2 \times 1/2}{1 + 1/4} = \frac{4}{5}$$

At point D

$$\frac{T_3}{\sin(180^\circ - \theta)} = \frac{T_2}{\sin 90^\circ} = \frac{T_4}{\sin(90^\circ - \theta)}$$



$$\Rightarrow T_3 = -T_2 \times \sin \theta = 2236 \times \frac{1}{\sqrt{5}} = 999.96 \text{ N}$$

$$T_4 = T_2 \times \cos \theta = -2236 \times \frac{2}{\sqrt{5}} = -1999.93 \text{ N}$$

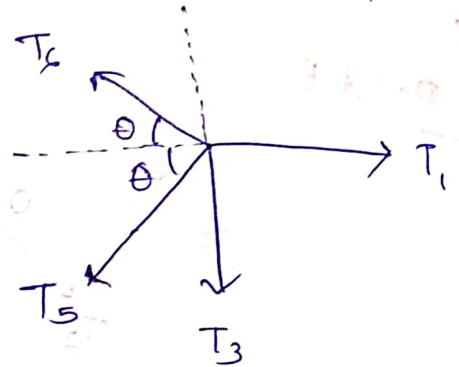
At point C

$$\sum F_x = 0$$

$$\Rightarrow T_c \cos \theta + T_s \cos \theta = T_1$$

$$\Rightarrow T_s + T_c = \frac{2000}{\cos \theta} = \frac{2000}{\frac{2}{\sqrt{5}}} \times \sqrt{5}$$

$$= 1000\sqrt{5} = 2236 \text{ N} \quad \text{--- (I)}$$



$$\sum F_y = 0$$

$$\Rightarrow T_3 + T_s \sin \theta = T_c \sin \theta$$

$$\Rightarrow 999.96 + T_s \times \frac{1}{\sqrt{5}} = T_c \times \frac{1}{\sqrt{5}}$$

$$\Rightarrow (T_s - T_c) = -999.96 \times \sqrt{5} = -2235.97 \text{ N} \quad \text{--- (II)}$$

Solving (I) & (II)

Method of Section:

Rules:

- (i) The section should be passed through the members and not through the joints.
- (ii) The section should divide the truss into 2 clearly separate and un-connected portions.
- (iii) A section should cut 3 members since only 3 unknowns can be determined from 3 eq^s of equilibrium.

Steps -

- (i) Calculate the support reaction (Note: if all supports reactions lie on one side of the section, then don't calculate the support reaction).
- (ii) Choose a point about which moment is to be taken so that you reach to one eqn^s & unknown situation, (Note: if all support reaction lie on one side choose the point on the sectⁿ where support reactⁿ are present and take moment about the).
- (iii) If the choose point lie on L.H.S of the sectⁿ, then select the forces which lie on R.H.S. of the sectⁿ and take moment (vice versa).
- (iv) Then consider other 2 equilibrium condition $\sum F_x = 0$ and $\sum F_y = 0$ on the section where the forces are consider.

Ex: find the axial forces DE, DB and AB of the given truss.

Solⁿ:

$$\sum F_x = 0$$

$$\Rightarrow H_A = 0$$

$$\sum F_y = 0$$

$$\Rightarrow R_A + R_C = 4 + 2$$

$$\Rightarrow R_A + R_C = 6 \quad \text{--- (1)}$$

$$\sum M_A = 0$$

$$\Rightarrow -R_C \times 6 + 4 \times (3 + 1.5) + 2 \times (4.5) = 0$$

$$\Rightarrow 6R_C = 4 \times 4.5 + 3$$

$$= 21.0$$

$$\Rightarrow R_C = \frac{7}{2} = 3.5 \text{ KN.}$$

$$R_A = 2.5 \text{ KN}$$

Joint B (R.H.S) :

Select forces on left hand side,

$$\sum M_B = 0$$

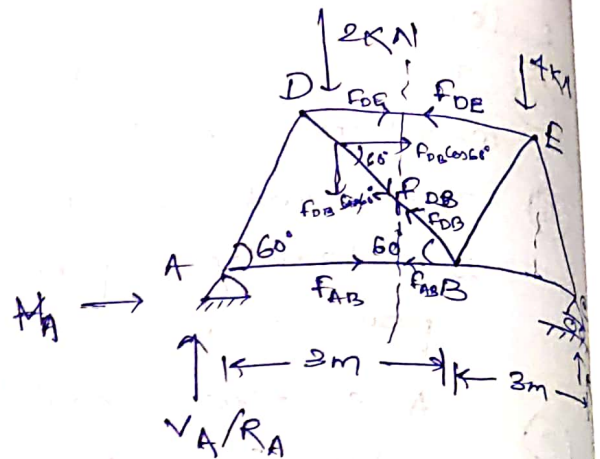
$$\Rightarrow 2.5 \times 3 + F_{DE} \times \frac{3\sqrt{3}}{2} = 2 \times 1.5$$

$$\Rightarrow F_{DE} = (3 - 7.5) \times \frac{2}{3\sqrt{3}} = -1.732 \text{ KN. (C)}$$

$$\sum F_x = 0 \quad (\text{L.H.S})$$

$$\Rightarrow F_{DB} \cos 60^\circ + F_{DE} + F_{AB} = 0$$

$$1.732 \times \frac{F_{DB}}{2} + (-1.732) + F_{AB} = 0$$



$$\sum F_y = 0 \quad (\text{L.H.S.})$$

$$\Rightarrow -2 + R_A - F_{DB} \sin 60^\circ = 0$$

$$\Rightarrow R_A = F_{DB} \times \frac{\sqrt{3}}{2} + 2$$

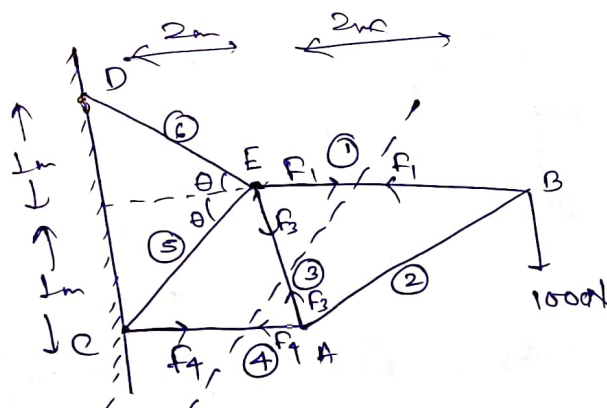
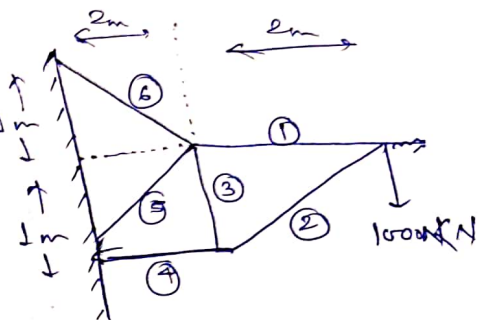
$$\Rightarrow F_{DB} = 0.577 \text{ kN}$$

$$\begin{aligned} \text{So } F_{AB} &= 1.22 - \frac{0.577}{2} \\ &= 1.4415 \text{ kN} \end{aligned}$$

Eg:

find axial forces along the member ①, ③, ④.

Soln:



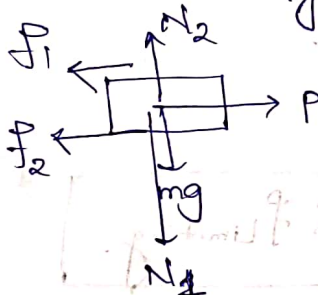
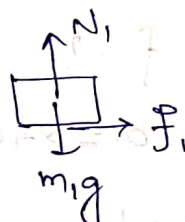
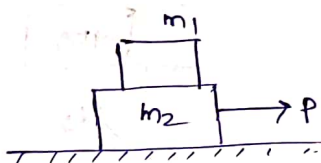
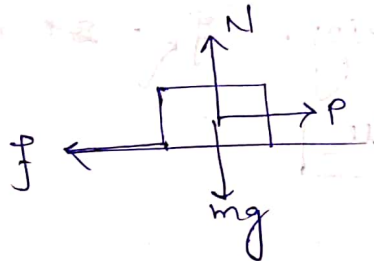
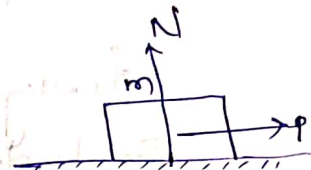
$$\sum M_E = 0 \quad (\text{Consider R.H.S.})$$

$$\Rightarrow F_4 \times 1 + 1000 \times 2 = 0$$

$$\Rightarrow F_4 = -2000 \text{ kN}$$

Module - 2 (Friction)

- It is contact force.
- Tangential to the surface.



Friction : ① Static, ② Kinetic ③ Rolling

Static (f_s)

- ① Body is in Rest.
- ② Variable in nature
- ③ $0 \leq f_s \leq f_{\text{limiting}}$
 $f_{\text{limiting}} = \mu_s N$

μ_s : coefficient of limiting static friction

Kinetic (f_k)

- ① Body is in motion.
- ② It is constant in nature.
- ③ $f_k \propto N$

$$f_k = \mu_k N$$

μ_k : coefficient of kinetic friction.

Rolling X
(Not in syllabus)

Kinetic friction:

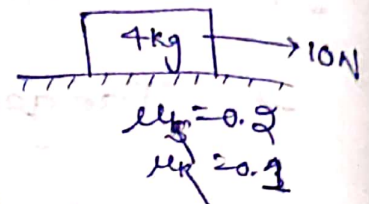
Eg: 1

find $f_k = ?$

$$f_{\text{limiting}} = 0.2 \times 40 = 8 \text{ N.}$$

So body is moving,

$$f_k = 0.1 \times 40 = 4 \text{ N.}$$



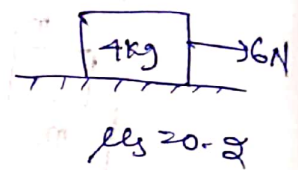
$$\mu_k < \mu_s$$

Eg: 2

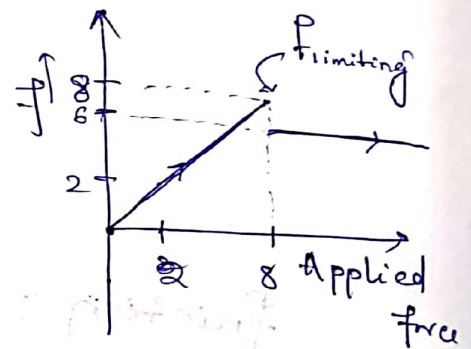
find $f = ?$

$$f_{\text{max}} = 0.2 \times 40 = 8 \text{ N.}$$

$$f = 6 \text{ N.}$$



$$0 \leq f_s \leq f_{\text{Limiting}}$$



Static friction -

Kinetic friction -

Angle of friction:-

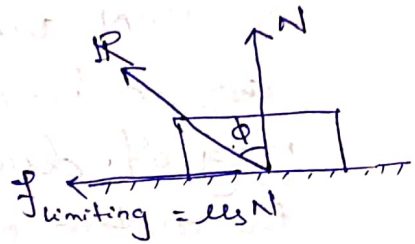
ϕ : Angle of friction.

It is only defined when ϕ is max.

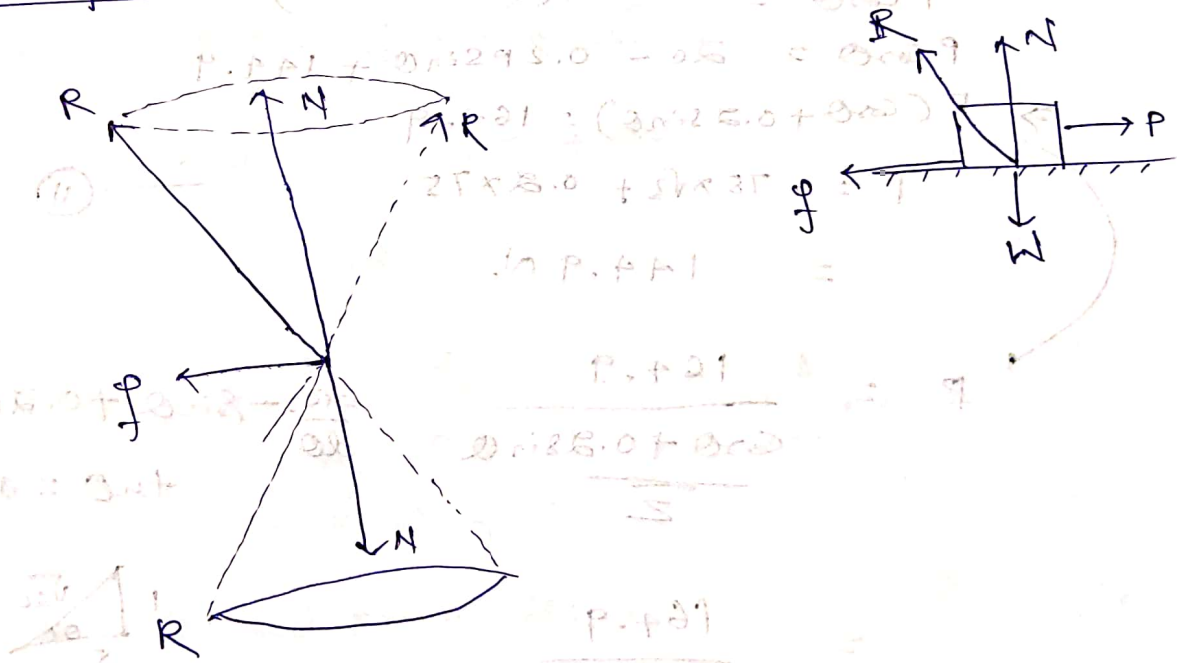
$$\tan \phi = \frac{f_l}{N} = \frac{\mu_s N}{N} = \mu_s$$

$$\Rightarrow \phi = \tan^{-1}(\mu_s)$$

Limiting friction:



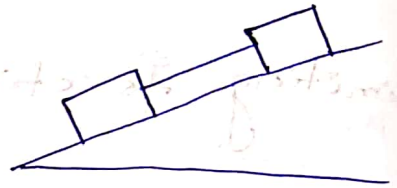
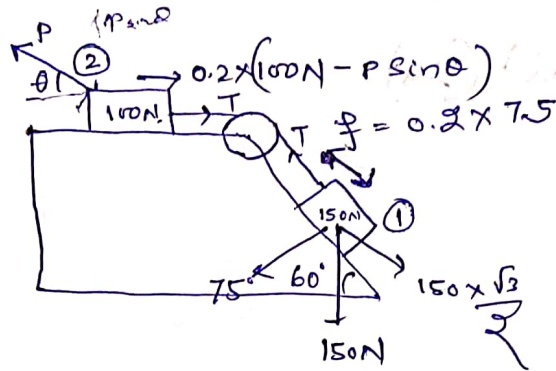
Cone of friction:



If the resultant reaction is rotated about the normal reaction force, it will form a cone known as cone of friction.

1. The rectangular boxes of w_1 is ~~20~~ 150 N and $w_2 = 100$ N are connected by a string and rest on an inclined plane; ~~at~~ ($\mu = 0.2$). Find the mag. of directⁿ of the least force P at which the motion of the block will impend?

Soln



$$P \cos \theta = 0.2(100 - P \sin \theta) + T \quad \text{--- (i)}$$

$$P \cos \theta = 20 - 0.2 P \sin \theta + 144.9$$

$$P(\cos \theta + 0.2 \sin \theta) = 164.9$$

$$T = 75 \times \sqrt{3} + 0.2 \times 75 = 144.9 \text{ N.}$$

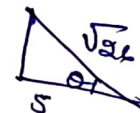
$$P = \frac{164.9}{\cos \theta + 0.2 \sin \theta}$$

$$\frac{dz}{d\theta} = -\sin \theta + 0.2 \cos \theta = 0$$

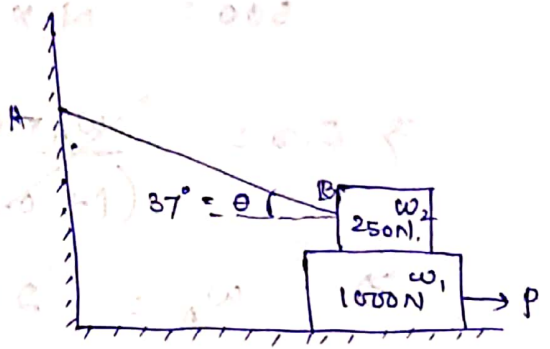
$$\tan \theta = 0.2 = \frac{1}{5}$$

$$= \frac{164.9}{\frac{5}{\sqrt{26}} + \frac{2}{10} \times \frac{1}{\sqrt{26}}}$$

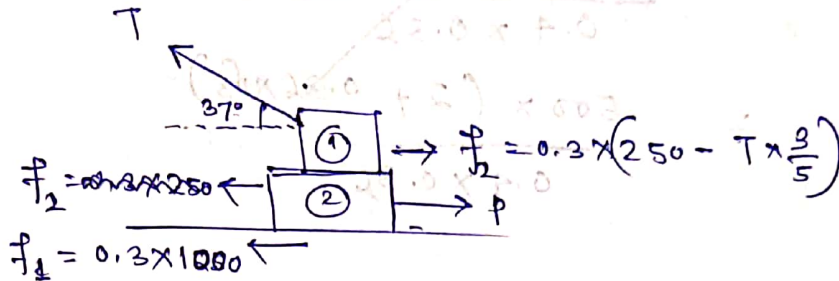
$$= 161.69 \text{ N.}$$



Q: $P = ?$ for impending motion?
($\mu = 0.3$)



Soln:



$$T \cos 37^\circ = 0.3 \times (250 - \frac{3T}{5})$$

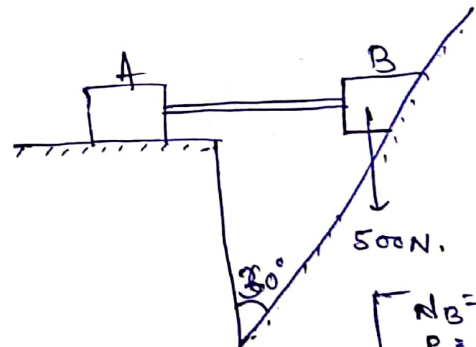
$$\frac{4T}{5} + \frac{9T}{50} = 75 \Rightarrow T = \frac{75}{\frac{4}{5} + \frac{9}{50}} = 76.53 \text{ N.}$$

$$P = 0.3 \times (250 - \frac{3T}{5}) + 0.3 \times (1000)$$

$$= 75 - \frac{9}{50} \times 76.53 + 300$$

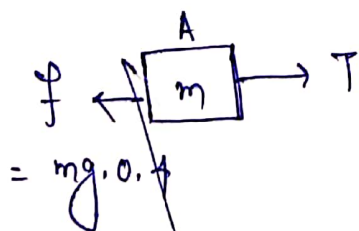
$$= 361.22 \text{ N.}$$

Q. Smallest wt. of A for equilibrium. $\mu_A = 0.4$,
Angle of friction for B on inclined plane $= 20^\circ$.



$$\begin{aligned} N_B &= 615.9 \text{ N} \\ P &= 422.5 \text{ N} \\ N_A &= 1056 \text{ N} \\ W_A &= 1656 \text{ N} \end{aligned}$$

$$\mu_B = \tan 20^\circ = 0.36$$

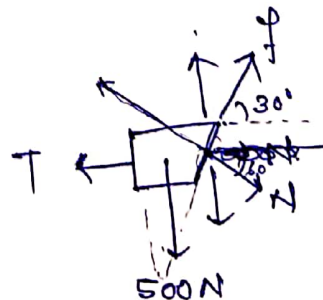


$$f = \mu \cdot N$$

$$T = \mu \cdot N$$

$$T = \frac{N}{2} + f \times \frac{\sqrt{3}}{2}$$

$$= N \left(\frac{1}{2} + 0.36 \times \frac{\sqrt{3}}{2} \right)$$



$$500 \sin 30^\circ = N (0.36) \times \frac{1}{2}$$

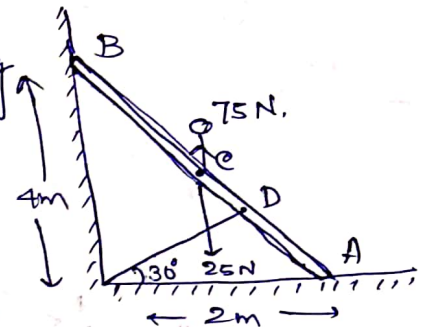
$$\Rightarrow 500 = \mu \cdot W_A \times 0.4 \times 0.36 \times \frac{1}{2}$$

$$\Rightarrow W_{tA} = \frac{500 \times 2}{0.4 \times 0.36} = 6944.44 \text{ N.}$$

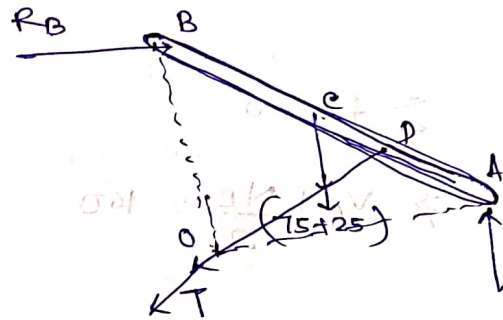
Ladder

1. Without friction.
2. With friction.

Q. A man wt.ing 75N stands on a middle of a ladder find the tension in the string & Reaction at point A & B as shown in the figure.



Soln:-



$$\begin{aligned} T &= 40.58 \text{ N} \\ R_B &= 35 \text{ N} \\ R_A &= 120.3 \text{ N} \end{aligned}$$

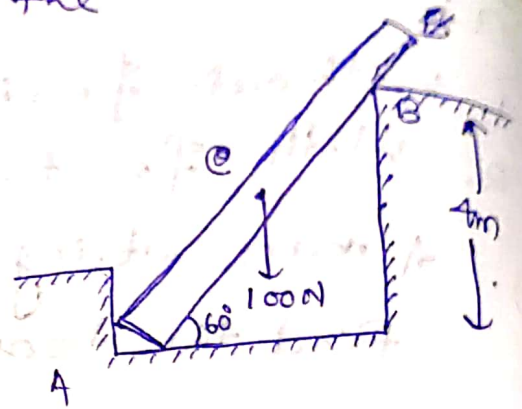
$$\sum F_x = 0$$

$$\sum F_y = 0$$

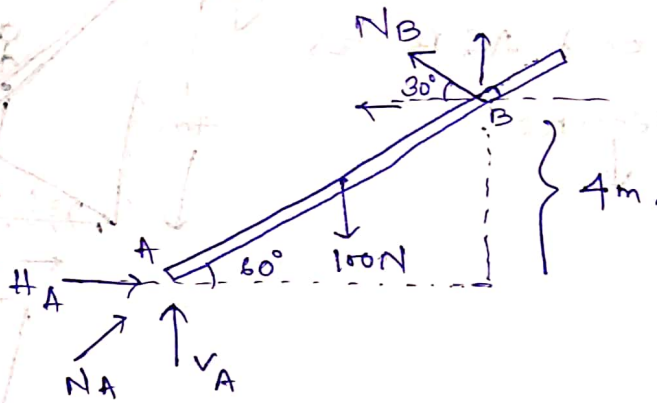
$$\sum M_o = 0$$

Q.2. A man wt. 100 N stands on the ladder of neglected wt.

Find reaction at A & B.
Neglect friction.



Solⁿ:



$$\sum f_x = 0$$

$$\Rightarrow H_A = N_B \cos 30^\circ$$

$$\Rightarrow H_A = \frac{\sqrt{3} N_B}{2} \quad \text{--- (1)}$$

$$\sum f_y = 0$$

$$\Rightarrow V_A + \frac{N_B}{2} = 100$$

$$\sum M_A = 0$$

$$\Rightarrow N_B \times \frac{8}{\sqrt{3}} = 100 \times \frac{2}{\sqrt{3}}$$

$$\Rightarrow N_B = \frac{200}{2} = 25 \text{ N.}$$

$$H_A = \frac{\sqrt{3} \times 25}{2} = 21.65 \text{ N}, \quad V_A = 100 - \frac{25}{2} = 87.5 \text{ N.}$$

$$N_A = \sqrt{H_A^2 + V_A^2} = 90.13 \text{ N.}$$

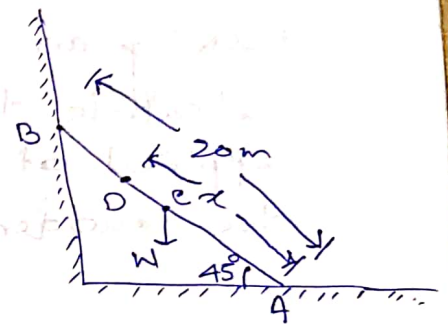
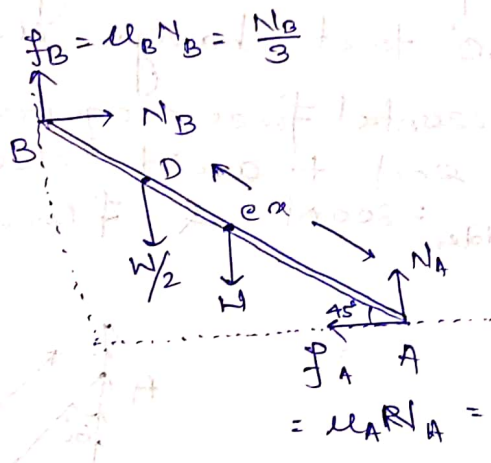
* Ladder with friction:

Q. 1 A uniform ladder AB of length 20m. & wt. W is supported by the horizontal road

It makes an angle 45° with horizontal. If a man whose wt. $W/2$ ascends the ladder, how much length (x) of the ladder he shall climb before ladder slip. ($\mu_B = 1/3$), ($\mu_A = 1/2$)

Q. 2. If a boy now stands on the end of the ladder what must be his ^{least force} ~~weight~~ so that the man may go on the top of the ladder.

Soln:-



$$\sum f_x = 0$$

$$\Rightarrow N_B = \frac{N_A}{2}$$

$$\sum f_y = 0$$

$$\Rightarrow \frac{N_B}{3} + N_A = W + \frac{W}{2} = \frac{3W}{2}$$

$$\Rightarrow \frac{N_B}{3} + 2N_B = \frac{7N_B}{3} = \frac{3W}{2}$$

$$\Rightarrow N_B = \frac{9W}{14}$$

$$\Rightarrow N_A = \frac{9W}{7}$$

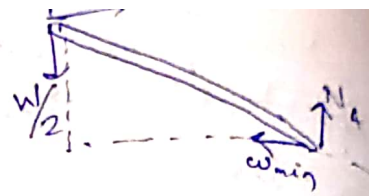
$$\sum M_A = 0$$

$$\Rightarrow N_B \times \frac{20}{\sqrt{2}} \neq \frac{N_B}{3} \times 20 \times \frac{1}{\sqrt{2}} = N \times \frac{1}{2} \times \frac{1}{\sqrt{2}} + \frac{W}{2} \times 20 \times \frac{1}{\sqrt{2}}$$

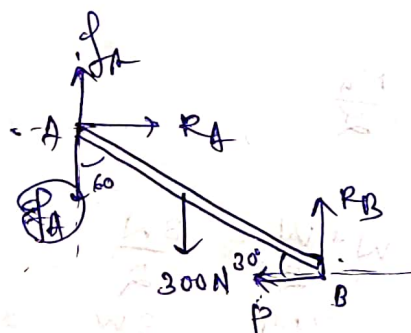
$$\Rightarrow \frac{9W}{14} \times \frac{20}{\sqrt{2}} + \frac{9W}{14 \times 3} \times 20 \times \frac{1}{\sqrt{2}} = \cancel{N} \times \frac{20}{2\sqrt{2}} + \cancel{W} \times \frac{20}{2\sqrt{2}}$$

$$\Rightarrow x = 2\sqrt{2} \left\{ \frac{9 \times 20}{14\sqrt{2}} + \frac{9 \times 20}{14 \times 3 \times \sqrt{2}} = \frac{20}{2\sqrt{2}} \right\}$$

$$= 14.28 \text{ m.}$$



- Q. The ladder length of 5m. is resting on a wall making an angle 60° to it heavy $\mu=0.3$. Calculate the horizontal force required to be applied at bottom end to avoid slipping of the ladder. $W_{\text{ladder}} = 300\text{N}$ & floor is smooth.



$\sum F_x = 0$

$$\sum F_x = 0 \Rightarrow R_A = P \quad \text{--- (i)}$$

$$\sum F_y = 0 \Rightarrow P_A + 300 = R_B \quad \text{--- (ii)}$$

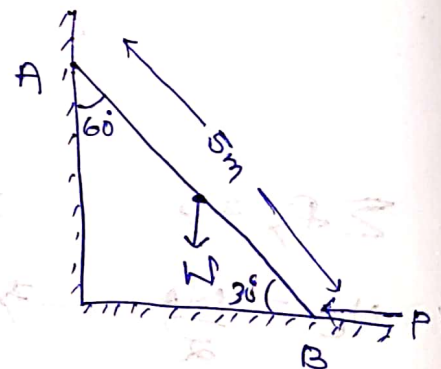
$$\Rightarrow 0.3 R_A + 300 = R_B$$

$$\sum M_B = 0$$

$$\Rightarrow 300 \times \frac{1}{2} \times 5 \times \frac{\sqrt{3}}{2} + 0.3 R_A \times \frac{\sqrt{3}}{2} \times 5 = R_A \times 5 \times \frac{1}{2}$$

$$\Rightarrow 75 \times 5 \times \sqrt{3} = R_A \left(\frac{5\sqrt{3}}{2} \times 0.3 + \frac{5}{2} \right)$$

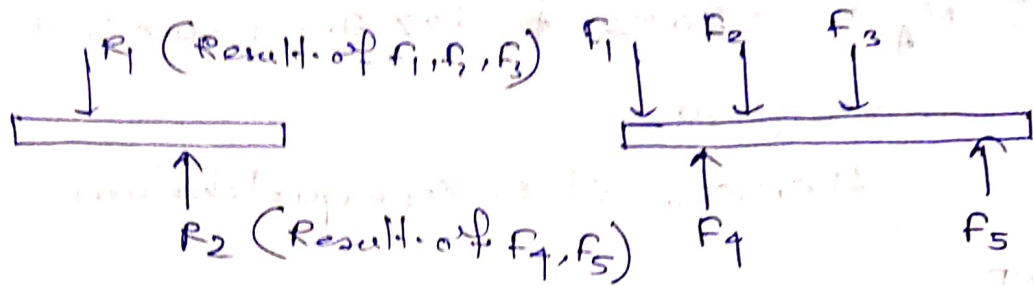
$$R_A = \frac{75 \times 5 \times \sqrt{3}}{\frac{5\sqrt{3}}{2} \times 0.3 + \frac{5}{2}} = 170\text{N}$$



$$[P = 170\text{N}]$$

Module-3 Parallel forces

* General Case of parallel forces:



Case - I

If $R_1 \neq R_2$ then the system is not in equilibrium

Case - II

If $R_1 = R_2$ & $\sum M_A \neq 0$, then the system is ~~not~~ in equilibrium with a couple.

Case III

If $R_1 = R_2$, $\sum M_A = 0$, the system is in equilibrium

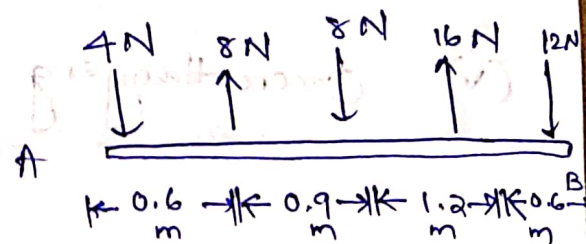
Eg - 1 Check its equilibrium.
Show

$$F_1 = 4 + 8 + 12 = 24 \text{ N}$$

$$F_2 = 8 + 16 = 24 \text{ N}$$

$$\begin{aligned}\sum M_A &= -8 \times 0.6 + 8 \times 1.4 \\ &\quad - 16 \times 2.6 + 12 \times 3.2 \\ &= 3.2 \neq 0.\end{aligned}$$

Hence it is not in equilibrium.

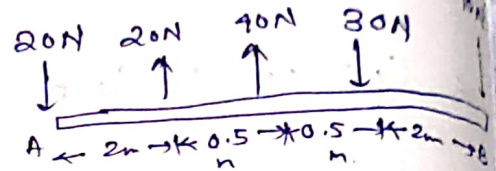


Exg: 2

$$R = -20 + 20 + 40 - 30 - 10 \\ = 0$$

$$\sum M_A = -20 \times 2 + 40 \times 2.5 + 30 \times 3 + 10 \times 5 \\ = 0$$

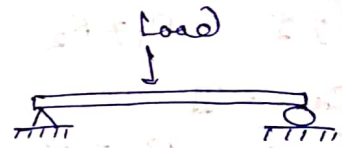
Hence it is in equilibrium.



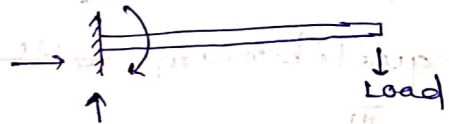
* Imp.

Types of Beam:

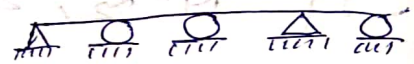
(i) Simply supported Beam



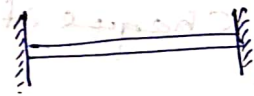
(ii) Cantilever Beam



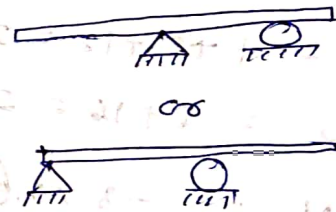
(iii) Continuous Beam



(iv) Fixed Beam

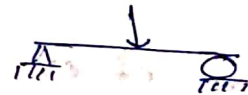


(v) Overhanging Beam

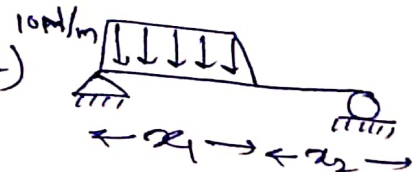


Types of loadings

(i) Point load



(ii) Continuous load / Uniformly distributed load (UDL)



(iii) Uniformly varying load (UVL)

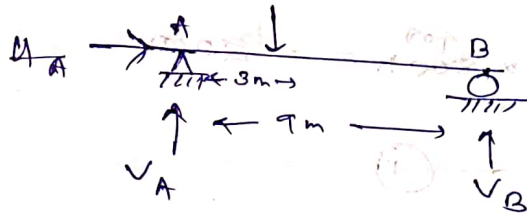
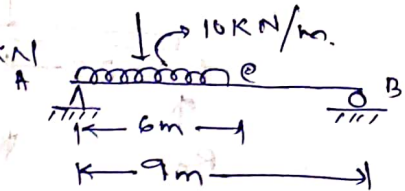
$$\text{Total load here} = \frac{1}{2} \times w \times L$$



Q. A simply supported beam of length 9m, carries a UDL of 10 kN/m for a distance of 6m from the left end. Calculate reactions at A & B.

Soln:-

Total load of UDL = $10 \times 6 = 60 \text{ kN}$
It will act at center of AC.



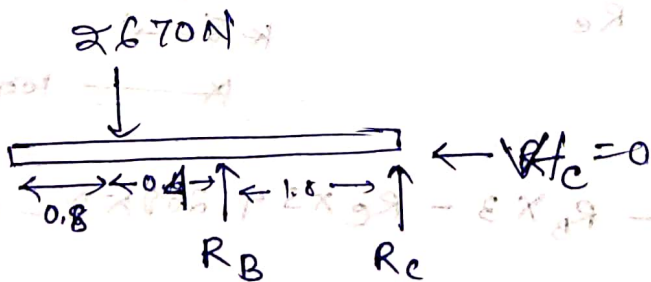
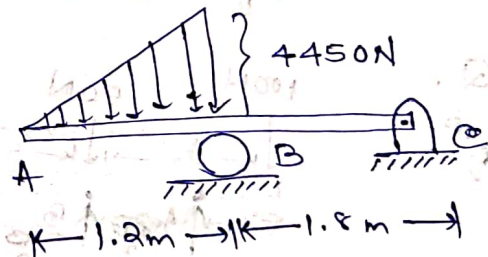
$$H_A = 0, \quad V_A + V_B = 60 \text{ kN.}$$

$$\sum M_A = 0 \Rightarrow V_A = (60 - 20) = 40.68 \text{ kN.}$$

$$\Rightarrow 60 \times 3 - V_B \times 9 = 0 \Rightarrow V_B = \frac{60 \times 3}{9} = 60/3 \text{ kN} = 20 \text{ kN.}$$

Q. Compute the support reaction.

Soln:-



$$\frac{1}{2} \times 4450 \times 1.2 = 2670 \text{ N}$$

$$\sum M = 0$$

$$\Rightarrow 2670 \times 2.2 = R_B \times 1.8$$

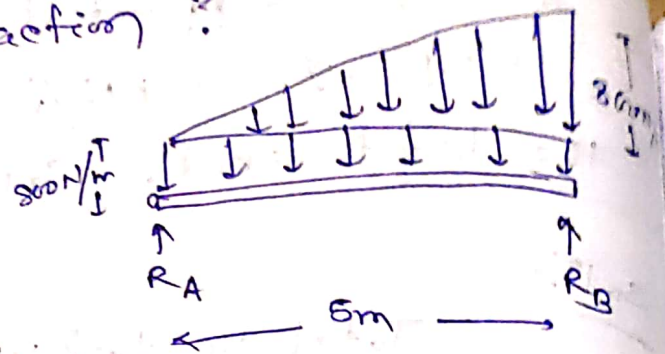
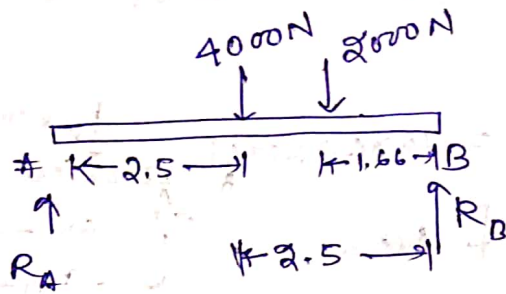
$$R_B = \frac{2670 \times 2.2}{1.8} = 3263.33 \text{ N.}$$

$$R_B + R_C = 2670 \quad \text{--- (i)}$$

$$R_C = 2670 - 3263.33 = -593.33 \text{ N.}$$

Q. find the support reaction ?

Soln:-



$$\frac{1}{2} \times 800 \times 5 = 2000$$

$$R_A + R_B = 6000 \text{ N} \quad \text{--- (1)}$$

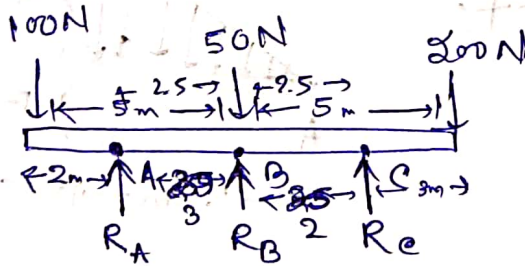
$$\sum M_B = 0$$

$$\Rightarrow 4000 \times 2.5 + 2000 \times 1.6 = R_A \times 5$$

$$\Rightarrow R_A = 2640 \text{ N}$$

$$R_B = 6000 - 2640 = 3360 \text{ N}$$

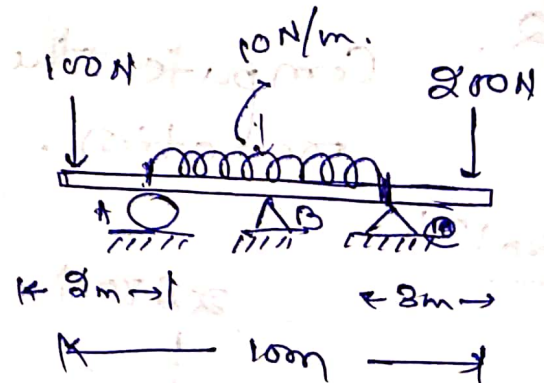
Q.



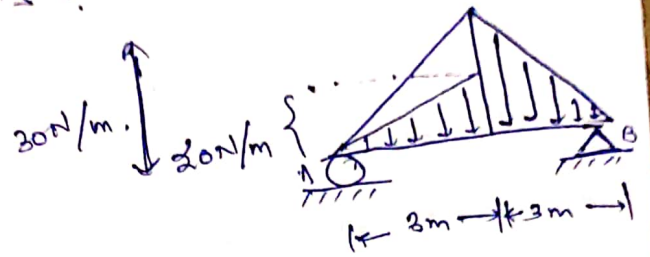
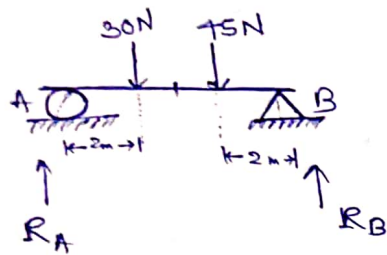
$$R_A + R_B + R_C = 350$$

$$\sum M_A = 0$$

$$\Rightarrow -100 \times 2 + 50 \times 2.5 - R_B \times 3 - R_C \times 5 + 200 \times 8 = 0$$



Q.2 find support reactions?



$$R_A + R_B = 75 \text{ N.}$$

$$\sum M_A = 0$$

$$\Rightarrow R_B \times 6 = 30 \times 2 + 45 \times 4$$

$$\Rightarrow R_B = \frac{60 + 180}{6} = \frac{240}{6} = 40 \text{ N.}$$

$$R_A = 75 - 40 = 35 \text{ N.}$$

Centroid / Center of Gravity / Center of Mass

C.G.: C.G. of a body is the point through which the whole wt. of the body 'x'

Centroid: The point at which the total area of the plane figure (like rectangle, sq. circle) is assumed to be concentrated is said to be centroid of the area.

Center of Mass:

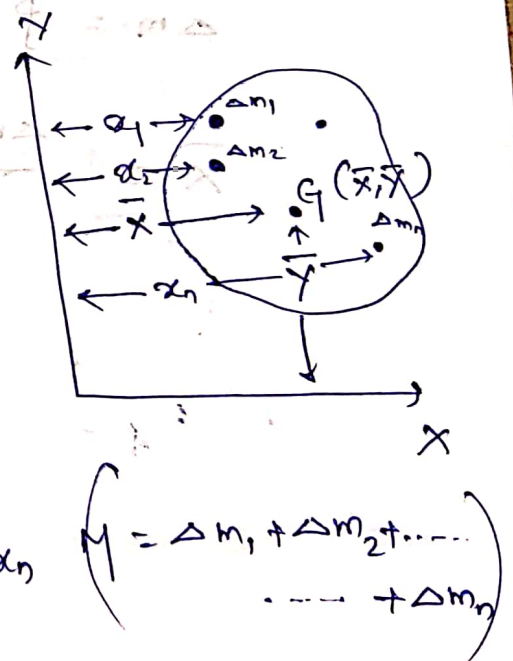
C.G of a body (By Method of Moment)

Moment of the resultant forces of all about Y-axis = Moment of all forces about Y axis

$$Mg \cdot \bar{X} = \Delta m_1 g x_1 + \Delta m_2 g x_2 + \dots + \Delta m_n g x_n$$

$$\bar{X} = \frac{\Delta m_1 x_1 + \Delta m_2 x_2 + \dots + \Delta m_n x_n}{M}$$

$$\bar{X} = \frac{\sum_{i=1}^n \Delta m_i x_i}{\sum_{i=1}^n \Delta m_i}$$

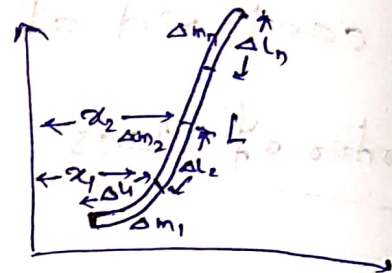


Similarly $\bar{y} = \frac{\sum \Delta m_i y_i}{\sum \Delta m_i}$

$$\bar{x} = \frac{\int dm_i x_i}{\int dm_i} \quad \bar{y} = \frac{\int dm_i y_i}{\int dm_i}$$

C.G of an one dimensional object:

Assumption - 1



1. Cross section of the wire is uniform through out the wire.
2. Material of the wire is homogeneous.

$\rho = M/V$ or $m = \rho V \Rightarrow \rho A \times l$

$$\Delta m = \rho A \Delta L$$

$$\bar{x} = \frac{\Delta m_1 x_1 + \Delta m_2 x_2 + \dots + \Delta m_n x_n}{\Delta m_1 + \Delta m_2 + \dots + \Delta m_n}$$

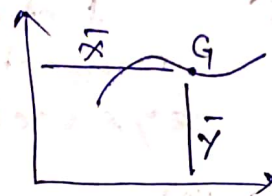
$$= \frac{(\rho A \Delta L_1 x_1) + (\rho A \Delta L_2 x_2) + \dots + (\rho A \Delta L_n x_n)}{\rho A \Delta L_1 + \rho A \Delta L_2 + \dots + \rho A \Delta L_n}$$

$$= \frac{\Delta L_1 x_1 + \Delta L_2 x_2 + \dots + \Delta L_n x_n}{\Delta L_1 + \Delta L_2 + \dots + \Delta L_n}$$

$$\boxed{\bar{x} = \frac{\sum \Delta L_i x_i}{\sum \Delta L_i}}$$

$$\boxed{\bar{y} = \frac{\sum \Delta L_i y_i}{\sum \Delta L_i}}$$

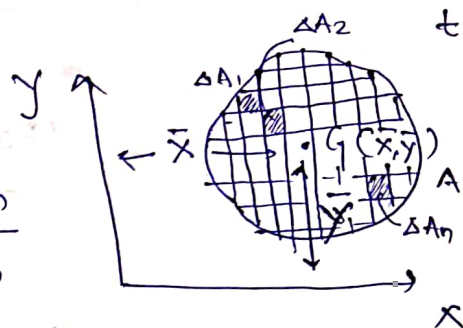
Q. $\bar{x} = \frac{\int dl \cdot x}{\int dl}$ $\bar{y} = \frac{\int dl \cdot y}{\int dl}$



C.G. of 2D object

$M = \int V = \int A \cdot t$

$\bar{x} = \frac{x_1 \Delta m_1 + x_2 \Delta m_2 + \dots + x_n \Delta m_n}{\Delta m_1 + \Delta m_2 + \dots + \Delta m_n}$



$(\Delta m = \rho \cdot \Delta A)$

$$\frac{\Delta A_1 x_1 + \Delta A_2 x_2 + \dots + \Delta A_n x_n}{\Delta A_1 + \Delta A_2 + \dots + \Delta A_n}$$

$$\bar{x} = \frac{\sum \Delta A_i x_i}{\sum \Delta A_i} = \frac{\int x dA}{\int dA}$$

$$\bar{y} = \frac{\sum \Delta A_i y_i}{\sum \Delta A_i} = \frac{\int y dA}{\int dA}$$

Notes

- ① The axis about which moments of areas are taken is known as axis of reference.
- ② If the given section is symmetrical about x-axis or y-axis then the C.G. of the section lie on the axis of symmetry.
- ③ C.G. doesn't depend on orientation of the body.

Q.1. Determine the centroid of cross-sectional area of an unequal eye section.

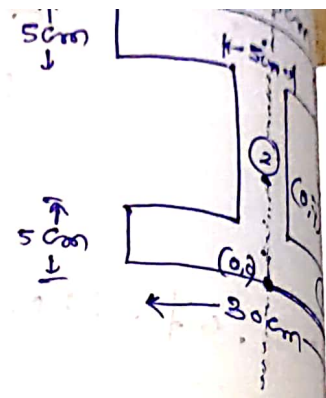
Soln:-

$$\bar{x} = 0$$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

$$= \frac{(30 \times 5) \times 2.5 + (15 \times 5) \times 12.5 + (20 \times 5) \times 22.5}{(30 \times 5) + (15 \times 5) + (20 \times 5)}$$

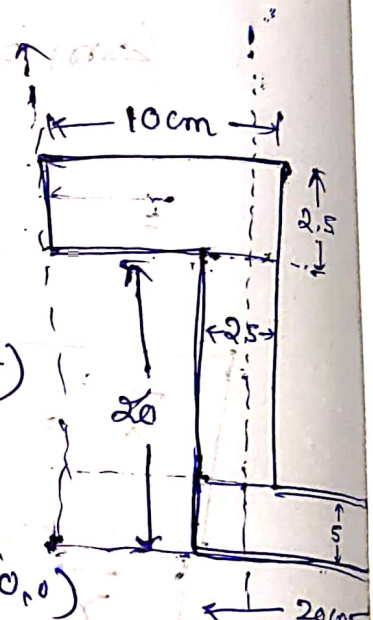
$$= 10.96 \text{ cm.}$$



Q.2.

$$\bar{x} = \frac{(10 \times 2.5) \times 5 + (2.5 \times 15) \times (10 - \frac{2.5}{2}) + (5 \times 20) \times (10 + 7.5)}{(10 \times 2.5) + (15 \times 2.5) + (20 \times 5)}$$

$$= 13.55 \text{ cm.}$$



$$\bar{y} = \frac{(10 \times 2.5) \times (24.25) + (2.5 \times 15) \times (12.5) + (5 \times 20) \times 2.5}{(10 \times 2.5) + (15 \times 2.5) + (20 \times 5)}$$

$$= 7.69 \text{ cm.}$$

Q. Find centroid of the given section?

$$\bar{x} = \frac{A_1 x_1 - A_2 x_2 + A_3 x_3}{A_1 - A_2 - A_3}$$

$$= \frac{(200 \times 150) \times 100 - \left(\frac{1}{2} \times 100 \times 75\right) \times \left(200 - \frac{100}{2}\right) - \pi \times (50)^2 \times 100}{(200 \times 150) - (50 \times 75) - \pi \times (50)^2}$$

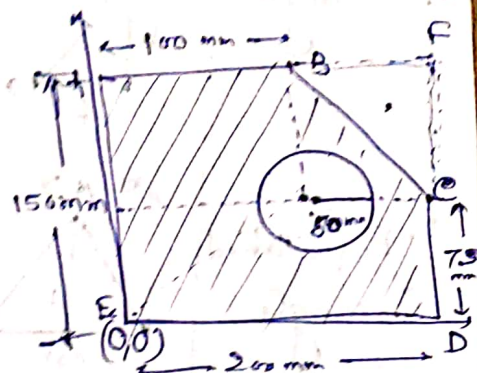
$$= \frac{(200 \times 150) \times 100 - (50 \times 75) \times 150 - \pi \times (50)^2 \times 100}{(200 \times 150) - (50 \times 75) - \pi \times (50)^2}$$

$$= 86.41 \text{ mm}$$

$$\bar{y} = \frac{(200 \times 150) \times 75 - (50 \times 75) \times \left(150 - \frac{75}{2}\right) - \pi \times (50)^2 \times 75}{(200 \times 150) - (50 \times 75) - \pi \times (50)^2}$$

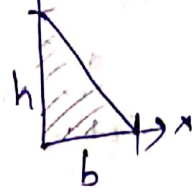
$$= \frac{(200 \times 150) \times 75 - (50 \times 75) \times 112.5 - \pi \times (50)^2 \times 75}{(200 \times 150) - (50 \times 75) - \pi \times (50)^2}$$

$$= 64.80 \text{ mm}$$



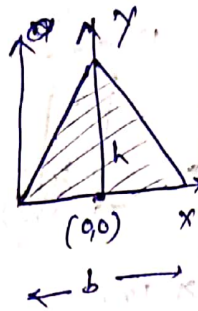
Description	Shape	Area	$\bar{x}$	$\bar{y}$
1. Rectangle		hb	$b/2$	$h/2$
2. Square		a^2	$a/2$	$a/2$

3. Triangle



$\frac{1}{2} \times b \times h$

4. Equilateral Triangle

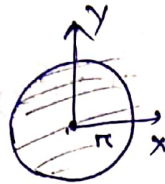


$$\frac{\sqrt{3}}{4} \times b^2$$

0

$\frac{h}{3}$

5. Circle

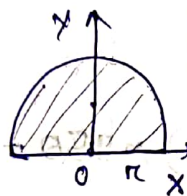


$$\pi r^2$$

0

0

7. Semi Circle

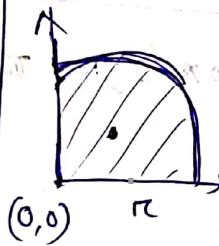


$$\frac{\pi r^2}{2}$$

0

$\frac{4\pi}{3\pi}$

8. Quarter circle



$$\frac{\pi r^2}{4}$$

$$\frac{4\pi}{3\pi}$$

$$\frac{4\pi}{3\pi}$$

Q. find out center of gravity of a right angle triangle by integration method.

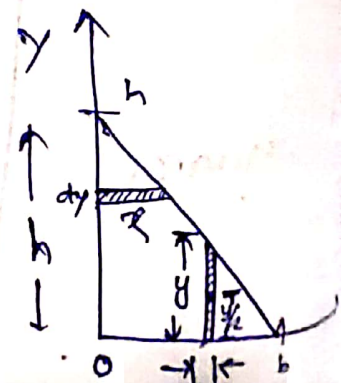
Soln:

$$\bar{x} = \frac{\int x dA}{\int dA}$$

$$\bar{y} = \frac{\int y dA}{\int dA}$$

$\bar{y} = ?$

$$= \frac{\int y dA}{\int dA} = \frac{\int_0^b \frac{y}{2} \cdot y \cdot dy}{\int_0^b y dy} = \frac{\int_0^b \frac{y^2}{2} dy}{\int_0^b y dy}$$



$$y = mx + c, \quad m = -h/b$$

$$y = -\frac{h}{b}x + h \Rightarrow \frac{hx}{b} = h - y \Rightarrow x = \frac{b}{h}(h - y)$$

$$\begin{aligned} \bar{y} &= \frac{\int_0^b (h - \frac{hx}{b})^2 dx}{bh} = \frac{\int_0^b h^2 dx + \frac{h^2}{b^2} \int_0^b x^2 dx - \frac{2h^2}{b} \int_0^b x dx}{bh} \\ &= \frac{h \left[b + \frac{1}{b^2} \times \frac{b^3}{3} - \frac{2}{b} \times \frac{b^2}{2} \right]}{b} \\ &= \frac{h}{b} \times \frac{b}{3} = h/3 \end{aligned}$$

$$\begin{aligned} \bar{x} &= \frac{\int_0^h \frac{x}{2} \cdot x \cdot dy}{\int_0^h x \cdot h \cdot dy} \\ &= \frac{\int_0^h x^2 dy}{hb} = \end{aligned}$$

$$\frac{\frac{b^2}{h^2} \int_0^h (h^2 + y^2 - 2hy) dy}{hb}$$

$$= \frac{b}{h^3} \left[h^2 \times h + \frac{b^3}{3} - 2h \times \frac{h^2}{2} \right]$$

$$= \frac{b}{h^3} \times h^3 \left[1 + \frac{1}{3} - 1 \right]$$

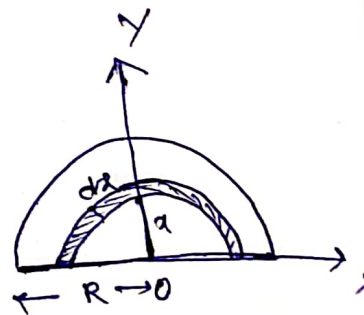
$$= \frac{b}{3}$$

Q. Find centroid of a semicircle by integration method.

soln:

$$\bar{y} = \frac{\int_0^R y dA}{\int_0^R dA} = \frac{\int_0^R \frac{2x}{\pi} \times x dx}{\frac{\pi R^2}{2}}$$

$$= \frac{\frac{2}{\pi} \int_0^R x^2 dx}{\frac{\pi R^2}{2}} = \frac{2}{\pi} \times \frac{R^3}{3} \times \frac{2}{\pi R^2} = \frac{4R}{3\pi}$$



$$\bar{y} = \frac{\int_0^R \frac{2x}{\pi} \cdot \pi x dx}{\int_0^R dA} = \frac{\int_0^R x^2 dx}{\frac{\pi R^2}{2}} = \frac{4R}{3\pi}$$

Assignment...

Q. Find out centroid of a quarter circle?

$$\bar{x} = \frac{\int dA \cdot x}{\int dA}$$

$$= \int_0^R \frac{\pi x}{2} \cdot dx \cdot \frac{2x}{\pi} \cdot \frac{1}{\sqrt{2}}$$

$$\bar{y} = \frac{\int_0^R \bar{y} dA}{\int_0^R dA}$$

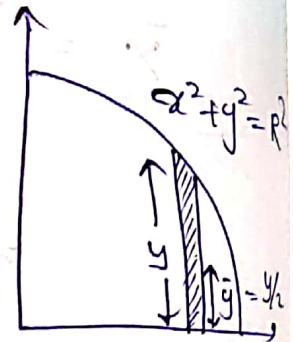
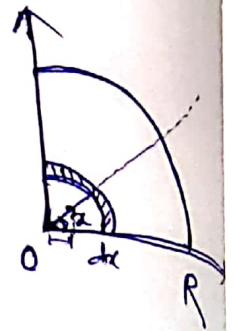
$$= \frac{\int_0^R \frac{y}{2} \cdot y dx}{\frac{\pi R^2}{4}}$$

$$= \frac{\frac{1}{2} \int_0^R y^2 dx}{\frac{\pi R^2}{4}} = \frac{\frac{1}{2} \int_0^R (R^2 - x^2) dx}{\frac{\pi R^2}{4}}$$

$$= \frac{2}{\pi R^2} \left[R^2 \int_0^R dx - \int_0^R x^2 dx \right]$$

$$= \frac{2}{\pi R^2} \left[R^2 \times R - \frac{R^3}{3} \right]$$

$$= \frac{4R^3}{3\pi R^2} = \frac{4R}{3\pi}$$



Q. Find out C.G. of an equilateral Δ ?

$$\bar{x} = 0$$

$$\bar{y} = \frac{\int_{-b/2}^{b/2} \frac{y}{2} \cdot y \cdot dx}{\int_{-b/2}^{b/2} dA}$$

$$= \frac{\int_0^{b/2} \frac{y^2}{2} \times dx}{\frac{\sqrt{3}}{4} \times b^2}$$

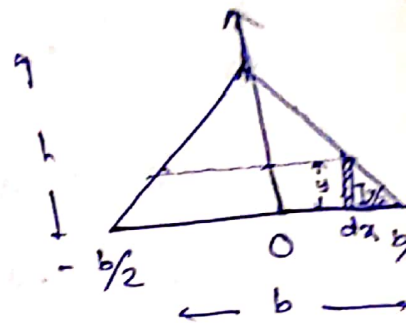
$$= \frac{\int_0^{b/2} h^2 \left(1 - \frac{2x}{b}\right)^2 dx}{\frac{\sqrt{3}}{4} b^2}$$

$$= \frac{h^2 \int_0^{b/2} \left[dx + \frac{4x^2}{b^2} dx - \frac{4x}{b} dx \right]}{\frac{\sqrt{3}}{4} b^2}$$

$$= \frac{h^2 \left[\frac{x}{1} + \frac{4}{b^2} \times \frac{b^3}{3 \times 2} - \frac{4}{b} \times \frac{b^2}{2 \times 2} \right]}{\frac{\sqrt{3}}{4} b^2}$$

$$= \frac{4h^2}{\sqrt{3}b^2} \left[\frac{b}{6} \right]$$

$$= \frac{4h^2}{\sqrt{3}b} \times \frac{1}{6} = \frac{4h^2}{6\sqrt{3}b} \rightarrow \times \frac{\sqrt{3}}{4\sqrt{3}}$$



$$y = mx + c$$

$$\Rightarrow y = \frac{-2h}{b}x + h$$

Q. find out centroid of a semiellipse by integration method?

Solⁿ:

$$\bar{x} = 0$$

$$\bar{y} = \frac{\int_{-a}^a \bar{y} dA}{\int_{-a}^a dA}$$

$$= \frac{\int_{-a}^a \frac{y}{2} \times y \cdot dx}{\frac{\pi ab}{2}}$$

$$= \frac{\frac{1}{2} \times \int_{-a}^a y^2 \cdot dx}{\frac{\pi ab}{2}} = \frac{2 \int_0^a \left(b^2 - \frac{x^2}{a^2}\right) dx}{\pi ab}$$

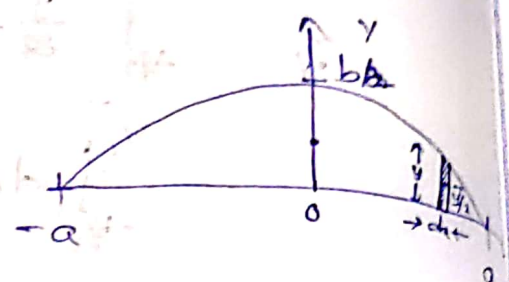
$$= \frac{2b^2 \left[xa - \frac{1}{3} \times \frac{a^3}{a^2} \right]}{\pi ab}$$

$$= \frac{2b^2 \left[1 - \frac{1}{3} \right]}{\pi ab}$$

$$= 2ab \times \frac{2}{3} \times \frac{1}{\pi}$$

$$= \frac{4ab}{3\pi}$$

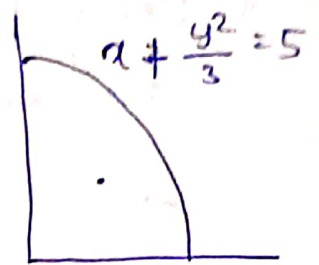
$$= \frac{4b}{3\pi}$$



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$y^2 = b^2 - \frac{x^2}{a^2}$$

find centroid?



Pappus Theorem: (Pappus & Guldinus Theorem)

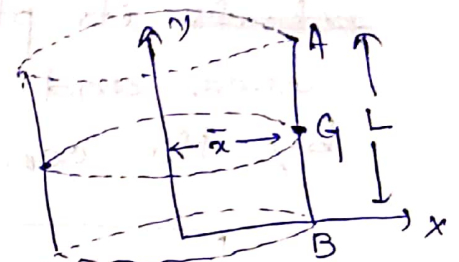
- ① The area of the surface generated by revolving a plane curve above a non-intersecting axis is equal to the product of length of the curve and the distance travelled by the centroid of curve during the revolution.

Area of the cylinder (cannel)

$$= L \times (\bar{x} \times \theta)$$

$$= L \times \pi \times 2\pi$$

$$= L \times 2\pi\pi$$

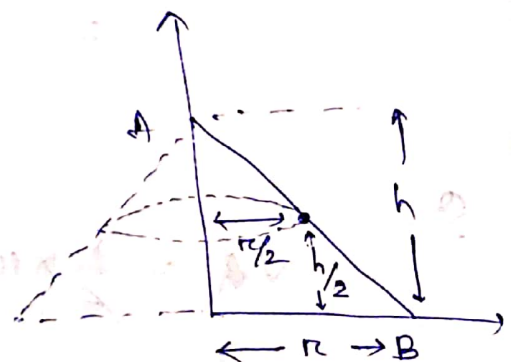


Ex: Find out the area?

Soln:-

$$A = \sqrt{h^2 + r^2} \times \frac{\pi}{2} \times 2\pi$$

$$= \pi r \sqrt{h^2 + r^2}$$



Ex: Find the centroid of the semicircular arc
if $A = \pi r^2$

Soln:-

$$2\pi r^2 = \frac{2\pi r}{2} \times \bar{x} \times 2\pi$$

$$\bar{x} = \frac{2r}{\pi}$$



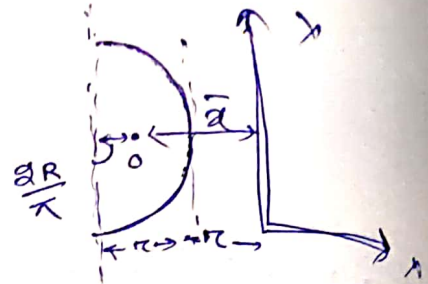
Q. Using Pappus theorem calculate the surface revolution shown in figure.

Soln:-

$$\bar{x} = 2R - \frac{2R}{\pi}$$

$$A = \frac{2\pi R}{2} \times \left(2R - \frac{2R}{\pi}\right) \times 2\pi$$

$$= 4\pi R^2 (\pi - 1)$$



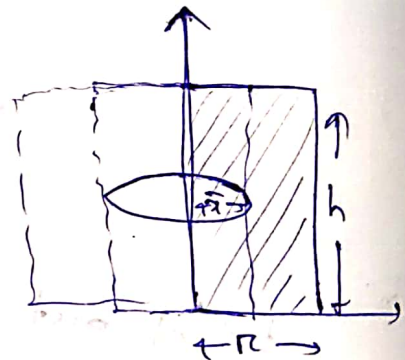
Pappus 2nd theorem:

- ① The volume of a solid generated by revolving a plane area ~~in~~ about non-intersecting axis ~~in~~ its plane is equal to the product of area and the length of the path travelled by the centroid of the area during revolution.

$$V = A \times (\bar{x} \times \theta)$$

$$= \pi h \times \frac{\pi}{2} \times 2\pi$$

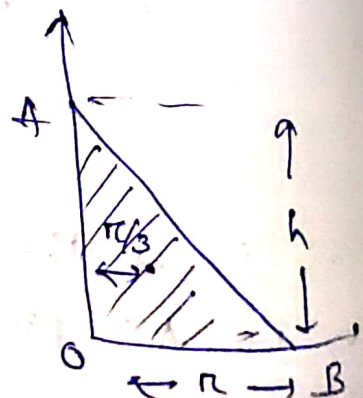
$$= \pi \pi^2 h$$



Q.

$$V = \frac{1}{2} \times \pi h \times \frac{\pi}{3} \times 2\pi$$

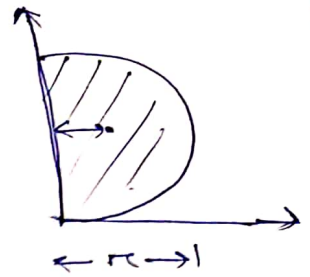
$$= \frac{1}{3} \pi \pi^2 h$$



Q.

$$V = \frac{\pi r^2}{2} \times \frac{4r}{2\pi} \times \cancel{2\pi}$$

$$= \frac{4}{3} \pi r^3$$

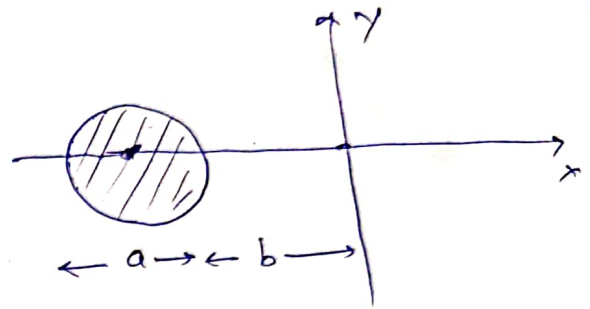


Q.

$$V = \frac{\pi a^2}{2} \times \left(b + \frac{a}{2}\right) \times \cancel{2\pi}$$

$$= \frac{\pi^2 a^2}{2} \times \frac{(2b+a)}{2}$$

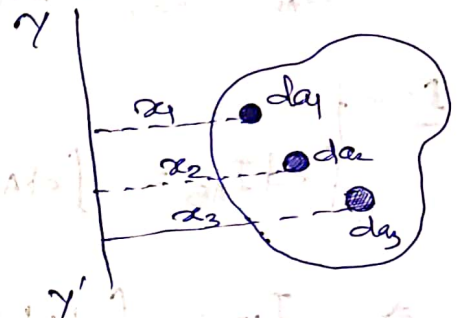
$$= \frac{\pi^2}{4} a^2 (2b+a)$$



Moment of Inertia (I)

$$I_{xx} = \int y^2 dA \quad I_{yy} = \int x^2 dA$$

$$I_{xx} = \sum_{i=1}^n y_i^2 dA_i \quad I_{yy} = \sum_{i=1}^n x_i^2 dA_i$$



* Perpendicular Axis Theorem: (Polar Axis theorem).

$$I_{xx} + I_{yy} = I_{zz}$$

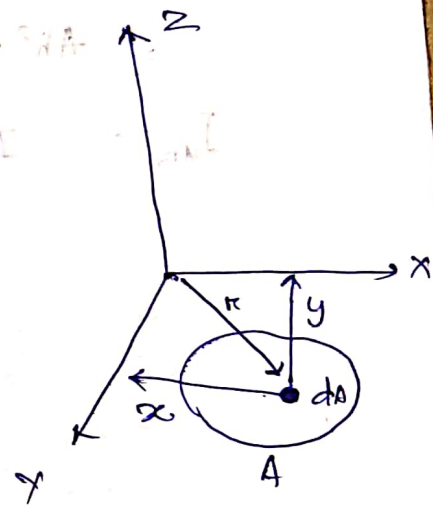
Proof: $r^2 = x^2 + y^2$

$$\int dI_{zz} = \int dA r^2$$

$$I_{zz} = \int dA (x^2 + y^2)$$

$$= \int x^2 dA + \int y^2 dA$$

$$= I_{yy} + I_{xx}$$



Condition: —

- (i) x, y, z axes are mutually perpendicular to each other.
- (ii) The body lies on xy plane and the area should be $\perp$ to z axis.

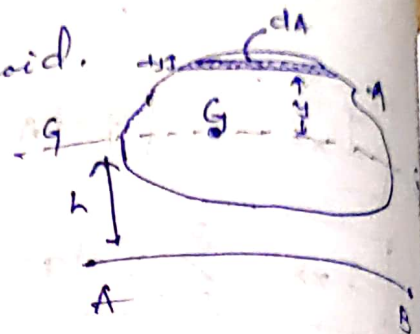


Parallel Axis theorem:

$I_{GG'} \rightarrow$ M.I. through centroid.

$GG' \parallel AB$

$$I_{AB} = I_{GG'} + Ah^2$$



Proof:-

$$\int dI_{AB} = \int dA(h+y)^2$$

$$\Rightarrow I_{AB} = \int dAh^2 + \int dAy^2 + \int dA2hy$$

$$= Ah^2 + I_G + 0$$

$$I_{AB} = I_G + Ah^2$$

Conditions -

(i) 2 axis must be parallel to each other

(ii) one axis must pass through the centroid of the body.

(2)

mass inertia $I = mr^2 : \text{kg-m}^2$ $I = ar^2 : \text{m}^4$ area inertia
--

*. Moment of inertia of different Section:

(1) Rectangular Section:-

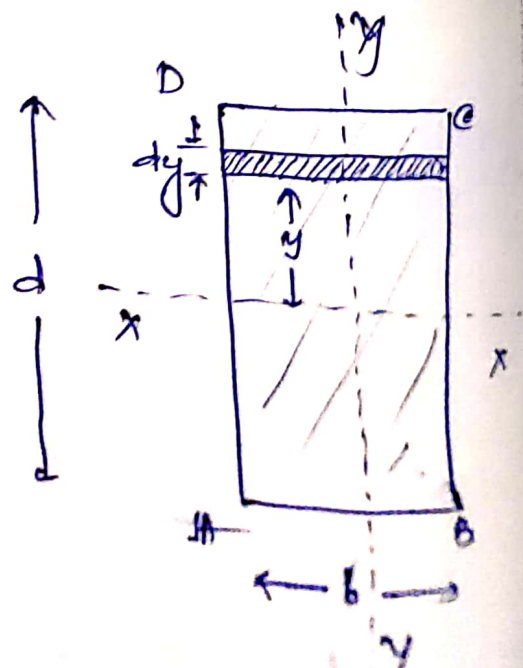
$$dI_{xx} = \int dAy^2$$

$$= 2 \int_0^{d/2} b \cdot dy \cdot y^2$$

$$= 2b \times \left[\frac{y^3}{3} \right]_0^{d/2}$$

$$= 2b \times \frac{1}{3} \times \frac{d^3}{8}$$

$$= \frac{bd^3}{12}$$



$$I_{xx} = \frac{bd^3}{12}, \quad I_{yy} = \frac{db^3}{12}$$

$$I_{xx} > I_{yy}$$

$$I_{AB} = I_G + A \cdot \left(\frac{d}{2}\right)^2 = \frac{bd^3}{3}$$

$$I_{AB} > I_{xx}$$

→ Moment of inertia is least about the centroidal axis:

(2) Moment of inertia of a circle:

$$I_{xx} = I_{yy}$$

$$\int dI_{zz} = \int dA \cdot r^2$$

$$\Rightarrow I_{zz} = \int_0^R 2\pi r dr r^2$$

$$= 2\pi \int_0^R r^3 dr = 2\pi \left[\frac{r^4}{4} \right]_0^R$$

$$= 2\pi \times \frac{R^4}{4} = \frac{\pi R^4}{2}$$

In terms of diameter $I_{zz} = \frac{\pi D^4}{32}$

$$I_{zz} = I_{xx} + I_{yy} = \frac{\pi D^4}{32}$$

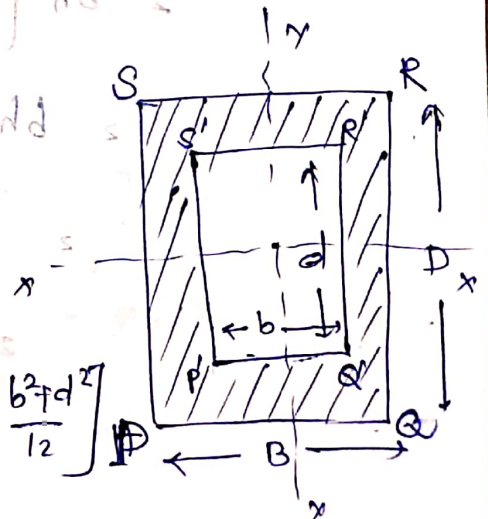
$$\Rightarrow 2I_{xx} = \frac{\pi D^4}{32} \Rightarrow I_{xx} = \frac{\pi D^4}{64} = I_{yy}$$

Q. Find Moment of inertia?

$$I_{xx} = \frac{BD^3}{12} - \frac{bd^3}{12}$$

$$I_{yy} = \frac{DB^3}{12} - \frac{db^3}{12}$$

$$I_{zz} = I_{xx} + I_{yy} = BD \left[\frac{D^2 + B^2}{12} \right] - bd \left[\frac{b^2 + d^2}{12} \right]$$

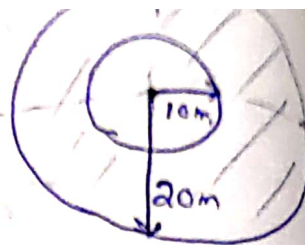


Q.2.

$$I_{zz} = 2I_{xx} \quad (\because I_{xx} = I_{yy})$$

$$= 2 \left[\frac{\pi (20)^4}{2} - \frac{\pi (10)^4}{2} \right]$$

$$= \pi [(20)^4 - (10)^4]$$

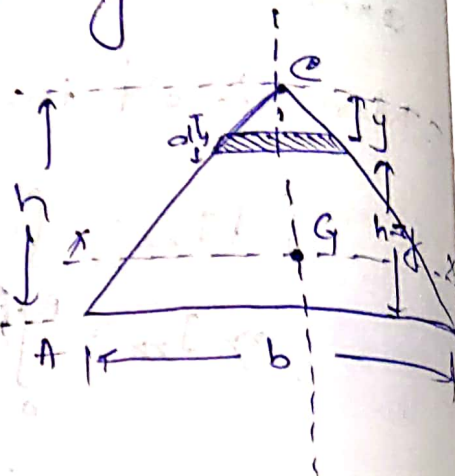


(3) Moment of inertia of a triangle section:-

$$\triangle ABC \cong \triangle PAC$$

$$\frac{PQ}{AB} = \frac{y}{h}$$

$$\Rightarrow \boxed{PQ = b \frac{y}{h}} \quad \text{--- (1)}$$



$$\int dI_{AB} = \int dA (h-y)^2$$

$$\Rightarrow I_{AB} = \int dy \cdot \frac{b \cdot y}{h} (h-y)^2$$

$$= \int y^2 \frac{b}{h} (h-y)^2 dy$$

$$= \int y^2 b h^2 dy + \int y^4 \frac{b}{h} dy - \int 2 y^3 b dy$$

$$= b h^2 \left[\frac{y^3}{3} \right]_0^h + \frac{b}{h} \left[\frac{y^5}{5} \right]_0^h - 2 b \left[\frac{y^4}{4} \right]_0^h$$

$$= b h^2 \cdot \frac{h^3}{3} + \frac{b}{h} \cdot \frac{h^5}{5} - 2 b \cdot \frac{h^4}{4}$$

$$= \frac{b h^5}{3} + \frac{b h^4}{5} - \frac{b h^4}{2}$$

$$= \frac{b h^5}{3} - \frac{3 b h^4}{10}$$

$$= b h^4 \left[\frac{h}{3} - \frac{3}{10} \right]$$

$$\left. \begin{aligned} & \frac{b h^5}{3} - \frac{3 b h^4}{10} \\ &= b h^4 \left[\frac{h}{3} - \frac{3}{10} \right] \end{aligned} \right\} \times I_{AB} = \frac{b h^3}{12}$$

$$I_{AB} = I_G + A\left(\frac{h}{3}\right)^2$$

$$\Rightarrow I_G = I_{AB} - A\left(\frac{h}{3}\right)^2$$

$$= \frac{bh^3}{12} - \frac{1}{2}bh \cdot \frac{h^2}{9}$$

$$I_{G_{xx}} = \frac{bh^3}{36}$$

$$I_{G_{yy}} = \frac{hb^3}{36}$$

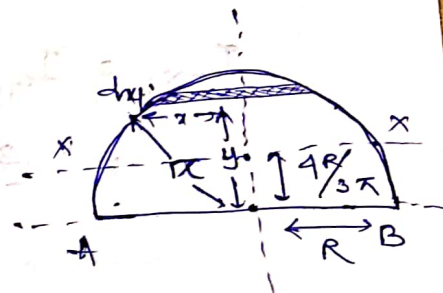
$$I_{AB} = \frac{bh^3}{12}$$

(4) M.I. of a semicircular Section:-

$$r^2 = x^2 + y^2 \Rightarrow 0 = 2x + 2y \frac{dy}{dx}$$

$$dI_{AB} = \int dA \cdot y^2$$

$$= \int dy \cdot 2x y^2$$



$$= 2x \sqrt{r^2 - x^2}$$

$$R \int_0^R 2 \sqrt{r^2 - y^2} \cdot y^2 dy = \frac{\pi R^4}{8}$$

$$I_{AB} = \frac{\pi R^4}{8} = \frac{\pi R^4}{8}$$

$$I_{G_{xx}} = \frac{\pi R^4}{8} - \frac{\pi R^2}{2} \times \frac{4R^2}{9\pi} = \frac{\pi R^4}{8} - \frac{16R^4}{918\pi}$$

$$= \frac{\pi R^4}{8} - \frac{8R^4}{9\pi} = R^4 \left[\frac{\pi}{8} - \frac{8}{9\pi} \right]$$

$$= 0.11 R^4$$

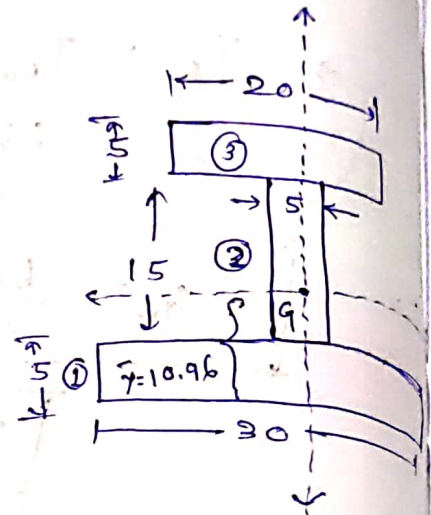
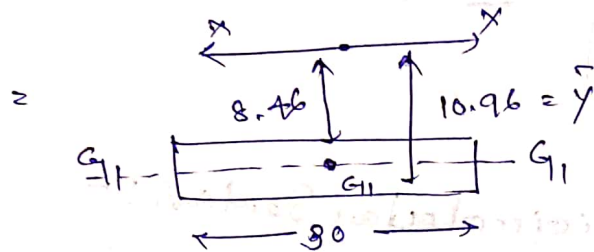
$$I_{G_{xx}} = 0.11 R^4$$

Q. Find the moment of inertia of the T section about the centroidal axis:

Soln:-

$$\bar{x} = 0, \quad \bar{y} = 10.96 \text{ cm.}$$

$$I_{xx} = I_{xx_1} + I_{xx_2} + I_{xx_3}$$



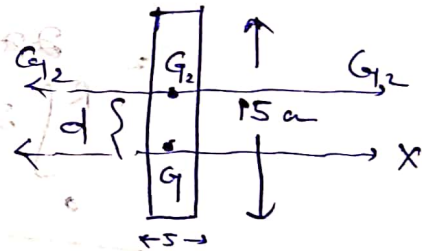
$$I_{xx_1} = I_{G_1 G_1} + A \bar{y}^2$$

$$= \frac{30 \times 5^3}{12} + (5 \times 30) \times 8.46^2 = 11648.24 \text{ cm}^4$$

$$I_{xx_2} = I_{G_2 G_2} + A d^2$$

$$= \frac{(15)^3 \times 5}{12} + (15 \times 5) \times (1.54)^2$$

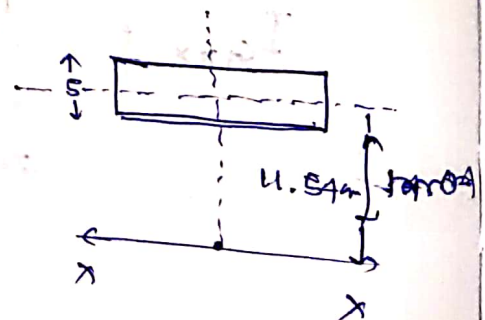
$$= 1584.12 \text{ cm}^4$$



$$I_{xx_3} = \frac{5^3 \times 20}{12} + (5 \times 20) \times (11.54)^2$$

$$= 13525.49 \text{ cm}^4$$

$$I_{xx} = 26157.85 \text{ cm}^4$$



Similarly $I_{yy} = I_{yy_1} + I_{yy_2} + I_{yy_3}$

$$I_{yy_1} = \frac{db^3}{12} = \frac{5 \times 30^3}{12}$$

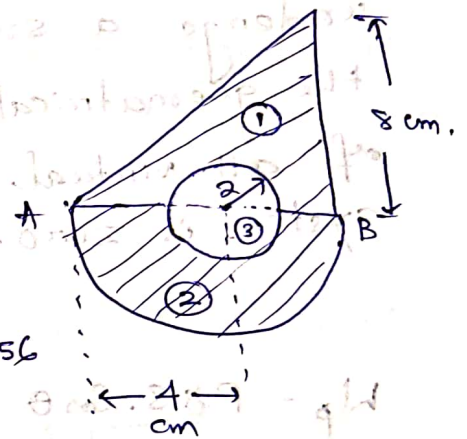
$$I_{yy_2} = \frac{db^3}{12} = \frac{15 \times 5^3}{12}$$

$$I_{yy_3} = \frac{db^3}{12} = \frac{5 \times 20^3}{12}$$

Q. Write the moment of Inertia of the shaded section w.r.t. AB.

$$I_{AB} = I_1 + I_2 - I_3$$

$$= \frac{8 \times 8^3}{12} + \frac{\pi \times 4^4}{8} - \frac{\pi \times 4^4}{8}$$



$$= 341.33 + 100.53 - 25.13$$

$$= 416.73 \text{ cm}^4$$

VIRTUAL WORK (Module-2)

VIRTUAL WORK: The work done by a force on a body due to small virtual displacement of a body is known as v.w.

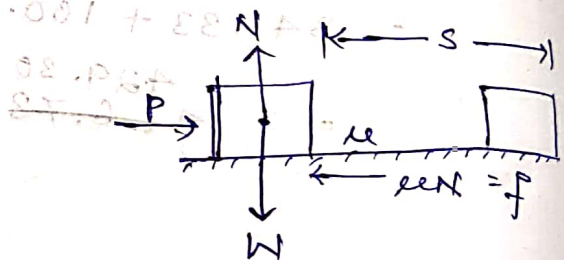
Principle of v.w: - It states if a system of forces acting on a body or a system of bodies be in equilibrium and if the system is imagined to undergo a small displacement consistent with the geometrical condition then the algebraic sum of all virtual work done by the forces by the system is zero.

$$W_p = P \times S \cdot \cos \theta = PS$$

$$W_w = W \cdot S \cdot \cos 90^\circ = 0$$

$$W_f = \mu N \cdot S \cdot \cos 180^\circ = -\mu NS$$

$$W_N = N \cdot S \cdot \cos 90^\circ = 0$$



Active forces: The forces for which v.w. is zero.

Non-Active / Passive forces: The forces for which v.w is not zero

According to principle of v.w $\sum W = 0$

$$\Rightarrow W_p + W_w + W_f + W_N = 0$$

W_{virtual} is true if force & disp. are in same direction or moment and displacement (or) torque is clockwise are in " " "

2)

→ forces to be omitted while applying v.w.

(i) Passive force i.e. force normal to direction of motion.

(ii) Reactions at smooth pins and hinges.

(iii) Tension produced in an inextensible string.

(iv) The wt. of a body when its c.g. moves in horizontal direction.

Q. A beam AB of span 10 meters carries 2 point loads of 15 kN and 20 kN at 4m & 6m from A respectively. Determine the beam reaction by principle of v.w.

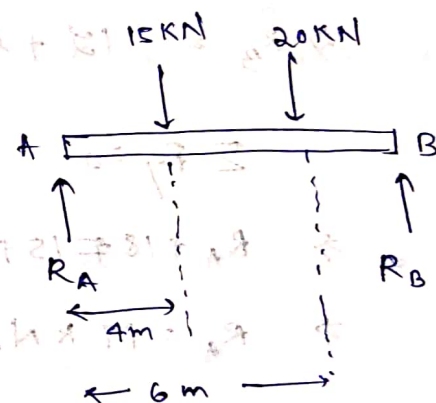
Soln: Method - 1

$$\sum M_A = 0$$

$$\Rightarrow -R_B \times 10 + 15 \times 4 + 20 \times 6 = 0$$

$$\Rightarrow R_B = \frac{60 + 120}{10} = \frac{180}{10}$$

$$= 18 \text{ kN}$$



$$\sum F_y = 0$$

$$\Rightarrow R_A + R_B = (15 + 20) \text{ kN}$$

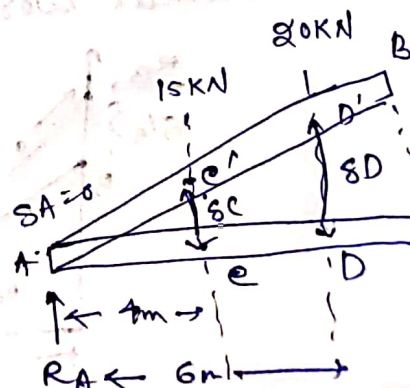
$$\Rightarrow R_A = 35 - 18 = 17 \text{ kN}$$

Method - 2

$$ACE' \sim ADD' \sim ABB'$$

$$\Rightarrow \frac{8_C}{4} = \frac{8_D}{6} = \frac{8_B}{10}$$

$$\Rightarrow \frac{8_C}{4} = \frac{8_D}{6} = \frac{8_B}{10}$$



$$\Rightarrow \delta_C = \frac{\delta_B \times 4}{10} = 0.4 \delta_B \quad \text{--- (1)}$$

$$\delta_D = \frac{\delta_B \times 6}{10} = 0.6 \delta_B \quad \text{--- (2)}$$

$$\text{V.W due to } R_B = R_B \delta_B$$

$$\text{V.W due to } R_D = -20 \times \delta_D$$

$$\text{V.W due to } R_C = -15 \times \delta_C$$

$$\text{V.W due to } R_A = 0$$

$$\sum W = 0$$

$$\Rightarrow R_B \delta_B - 20 \delta_D - 15 \delta_C = 0$$

$$\Rightarrow R_B \delta_B = 20 \times 0.6 \delta_B + 15 \times 0.4 \delta_B$$

$$\Rightarrow R_B = 12 + 6 = 18 \text{ kN.}$$

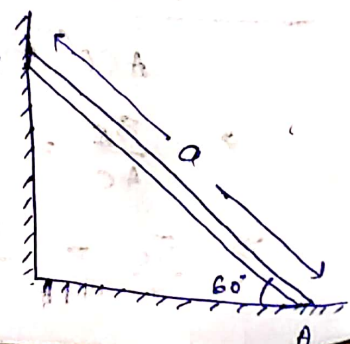
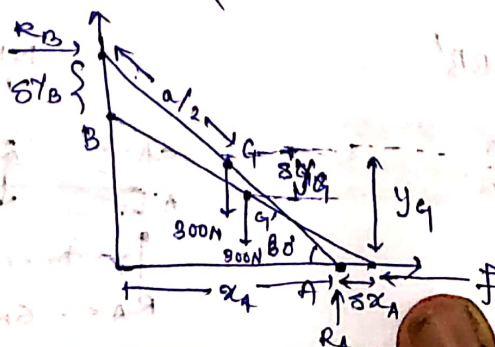
$$\sum F_y = 0$$

$$\Rightarrow R_A + 18 = 15 + 20$$

$$\Rightarrow R_A = 17 \text{ kN.}$$

Q.

A uniform ladder of wt. 300 N rests against a smooth vertical wall and a rough horizontal floor, making an angle 60° with horizontal, find the force of friction using the method of virtual work.



$$y_G = \frac{a}{2} \sin \theta = \frac{\sqrt{3}a}{4}$$

$$x_A = a \cos \theta = \frac{a}{2}$$

$$\Rightarrow |\delta x_A| = -\frac{a}{2} \sin \theta \quad \& \quad |\delta y_G| = \frac{a}{2} \cos \theta$$

$$- \text{V.W due to } R_A = 0$$

$$\text{V.W due to } R_B = 0$$

$$\text{V.W due to } f = -f \cdot \delta x_A$$

$$\text{V.W due to } 300 \text{ N} = 300 \times \delta y_G$$

$$\Sigma W_v = 0$$

$$\Rightarrow 300 \times \delta y_G = f \delta x_A$$

$$\Rightarrow 300 \times \frac{150}{2} \sin \theta = f \times \frac{a}{2} \cos \theta$$

$$\Rightarrow f = 150 \cot \theta = 150 \cot 60^\circ = \frac{150}{\sqrt{3}} = 86.6 \text{ N}$$

Q. Find Relationship betn P & Q ?

$$x_C = 2a \cos \theta$$

$$x_B = 6a \cos \theta$$

$$|\delta x_B| = 6a \sin \theta$$

$$|\delta x_C| = 2a \sin \theta$$

$$\text{V.W due to } Q = -Q \delta x_B$$

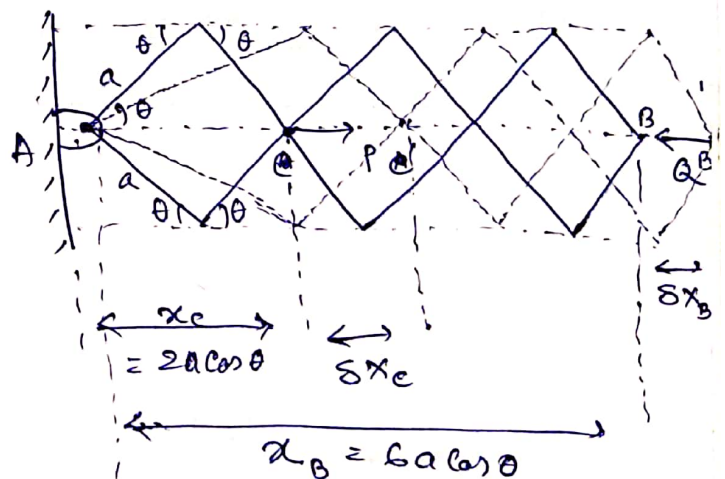
$$\text{V.W due to } P = P \delta x_C$$

$$\Sigma W_v = 0$$

$$\Rightarrow P \delta x_C = Q \delta x_B$$

$$\Rightarrow P \times 2a \sin \theta = Q \times 6a \sin \theta$$

$$\Rightarrow \boxed{P = 3Q}$$



Q. Calculate the moment M required to maintain the equilibrium?

$$x_c = 6 \cos \theta$$

$$(\delta x_c) = 6 \sin \theta d\theta$$

$$y_A = 4 \sin \theta$$

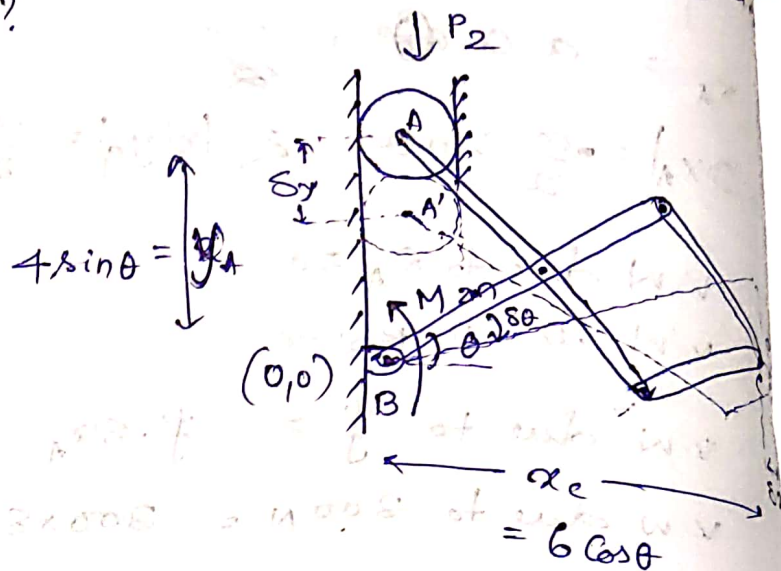
$$(\delta y_A) = 4 \cos \theta d\theta$$

$$\sum W_v = 0$$

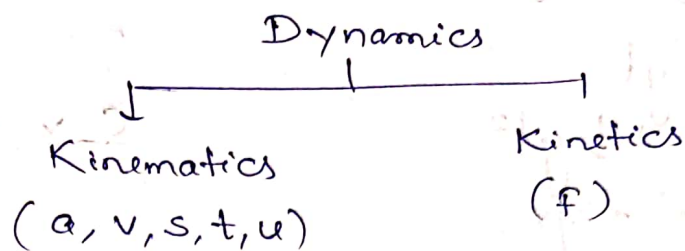
$$\Rightarrow P_1 \delta x_c + P_2 \delta y_A - M d\theta = 0$$

$$\Rightarrow P_1 6 \sin \theta d\theta + P_2 4 \cos \theta d\theta = M d\theta$$

$$\Rightarrow M = 6 P_1 \sin \theta + 4 P_2 \cos \theta$$



Rectilinear Translation (Module-4)



→ displacement $\leq$ distance i.e. $\vec{s} \leq d$

→ speed = d/t , velocity = $\frac{\vec{s}}{t}$, $\vec{v} \leq s$

→ acceleration $\vec{a} = \frac{\vec{v}}{t}$

Eqⁿ of Motion:

① $v = u + at$

② $s = ut + \frac{1}{2}at^2$

③ $v^2 = u^2 + 2as$

Q. A car is moving with a velocity of 15 m/s. The car is brought to rest by applying brake in 5 s. Find the a ?, distance?

Solⁿ: $v = 15 \text{ m/s}$, $u = 0$, $t = 5 \text{ s}$.

$$v = u + at$$

$$\Rightarrow 15 = 0 + a \cdot 5 \Rightarrow a = -3 \text{ m/s}^2 \text{ (retardation.)}$$

$$v^2 = u^2 + 2as$$

$$\Rightarrow 0 = -2 \cdot (-3) \cdot s$$

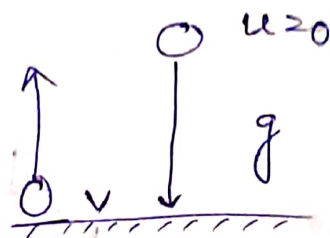
$$\Rightarrow s = \frac{225}{6} = 37.5$$

Freely falling body:

$$v = u + gt$$

$$s = ut + \frac{1}{2}gt^2$$

$$v^2 = u^2 + 2gs$$



Principles of Dynamics:

- (1) D'Alembert Principle $\rightarrow$ 2nd law of motion ($F = ma$)
- (2) Work-Energy principle $\rightarrow (W = K.E_2 - K.E_1)$
- (3) Impulse Momentum $\rightarrow$ Law of conservation of momentum.

* (i) D'Alembert Principle:

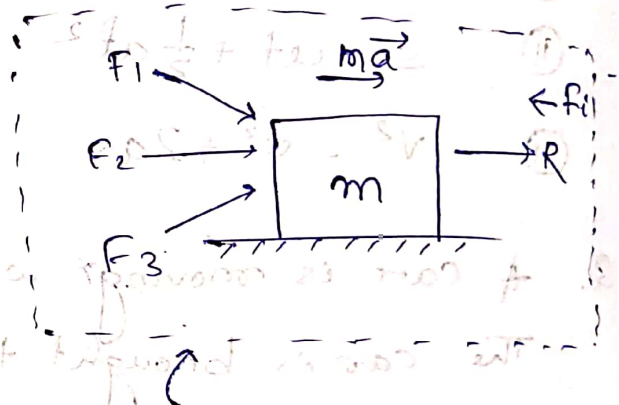
f_i = inertial force

$$f_i = m\vec{a}$$

D'Alembert rule $\rightarrow$

$$\sum F - f_i = 0$$

A system of forces acting on a body in motion is in dynamic equilibrium with the inertial force.



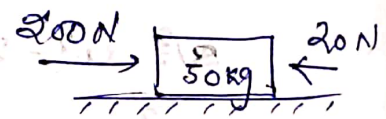
system is in dynamic equilibrium.

Eg: Find a ?

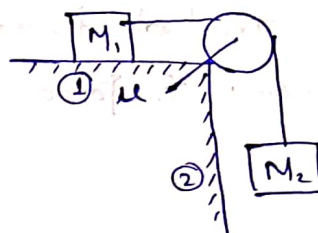
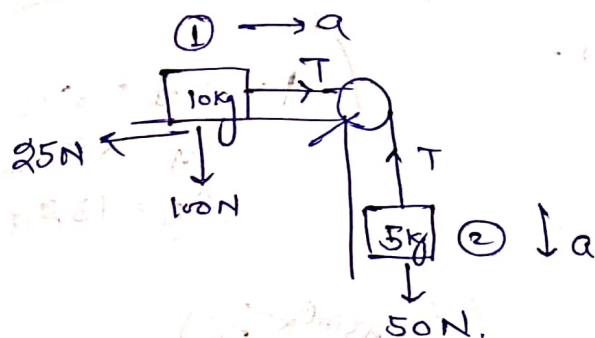
$$\sum F - f_i = 0$$

$$\Rightarrow 180 - 50a = 0$$

$$\Rightarrow a = \frac{180}{50} = 3.6 \text{ m/s}^2$$



Q. 2 blocks of mass m_1 and m_2 are connected by a flexible and inextensible string as shown in fig. μ for M_1 & floor is 0.25, find the a_1 & a_2 and $T = ?$ ($M_1 = 10\text{ kg}$, $M_2 = 5\text{ kg}$)



for block ①, $T - 25 = 10a$ — (i)

② $50 - T = 5a$ — (ii)

$$15a = 25$$

$$a_1 = a_2 = a = \frac{5}{3} = 1.67 \text{ m/s}^2$$

$$T = 10 \times \frac{5}{3} + 25 = \frac{75 + 50}{3} = \frac{125}{3} = 41.67 \text{ N}$$

Q. 2 blocks A and B of masses 5 kg and 20 kg are connected by an inclined string. A horizontal force of 100 N is applied to the block B. calculate T , $a = ?$ $\mu_A = 0.5$, $\mu_B = 0.25$

Forces towards direction of motion are +ve, and opposite to the direction of motions are -ve. (Sign convention.)

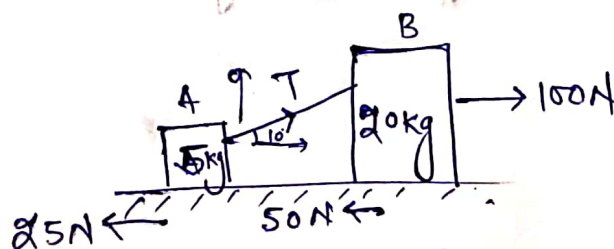
Soln: - $a = \frac{100 - 75}{25}$

$$= \frac{25}{25} = 1 \text{ m/s}^2$$

for Block A,

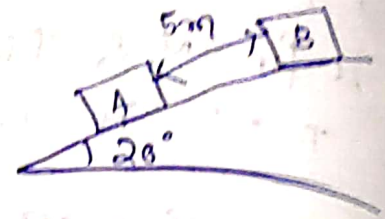
$$T - 25 = 5 \times 1 = 5$$

$$T = 25 + 5 = 30 \text{ N} \Rightarrow 30.46 \text{ N}$$



(Ans: $a = 1 \text{ m/s}^2$, $T = 30.46 \text{ N}$)

11.10 Q blocks A & B are held on an inclined plane 5m apart. $\mu_A = 0.2$, $\mu_B = 0.1$, If the blocks begins to slide down the plane simultaneously calculate the time and distance travelled by the block before collision.

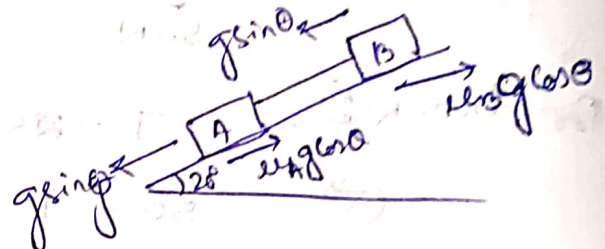


Ans: $t = 3.33$
 $S_A = 8.22m$
 $S_B = 13.22m$

Soln:- suppose
 At time t the collide.

$$S_B - S_A = 5$$

$$\Rightarrow \left[\frac{1}{2} \times g(\sin\theta - \mu_B \cos\theta) - \frac{1}{2} g(\sin\theta - \mu_A \cos\theta) \right] t^2 = 5$$



$$\Rightarrow t^2 = \frac{5 \times 2}{g \{ \sin\theta - \mu_B \cos\theta - \sin\theta + \mu_A \cos\theta \}}$$

$$= \frac{10}{9.8 \{ \mu_A - \mu_B \} \cos 20^\circ} = 10.858$$

$$\Rightarrow t = 3.3s$$

$$S_B = \frac{1}{2} \times g (\sin 20^\circ - 0.1 \cos 20^\circ) \times (3.3)^2 = 8.22m$$

$$S_A = \frac{1}{2} \times g (\sin 20^\circ - 0.2 \cos 20^\circ) \times (3.3)^2 = 13.22m$$

* (2) Work Energy Principle:

The work done by a force acting on a body during its displacement is equal to the K.E of the body during that displacement.

Proof:

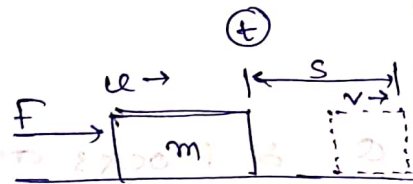
$$v^2 - u^2 = 2as$$

$$\Rightarrow \frac{m}{2} (v^2 - u^2) = 2as \times \frac{m}{2}$$

$$\Rightarrow \frac{1}{2} mv^2 - \frac{1}{2} mu^2 = mas$$

$$\Rightarrow F \cdot s = K.E_f - K.E_i$$

$$\Rightarrow W = K.E_f - K.E_i$$

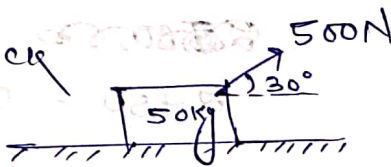


Note -

- ① If direction of force and displacement are same then W is +ve.
- ② If direction of force and displacement are opposite, W is -ve.

Q. A force of 500N, is acting on a mass 50kg.

Find velocity after the block has traveled a distance 10m. ($\mu = 0.5$)



Soln:

$$FS = K.E_f - K.E_i$$

$$\Rightarrow 500 \times \cos 30^\circ \times 10 = \frac{1}{2} \times 50 \times v^2$$

$$= 500 \times 0.5 \times 10 \times \frac{1}{2}$$

$$\Rightarrow \frac{1}{2} v^2 = \frac{20}{50} \times 10 \left(\frac{\sqrt{3}}{2} - 0.5 \right)$$

$$\Rightarrow v = 8.55 \text{ m/s}$$

$$\Rightarrow \left\{ 500 \times \cos 30^\circ - \left(500 - 500 \times \frac{1}{2} \right) \times 0.5 \right\} 10 = \frac{1}{2} \times 500 \times v^2$$

$$\Rightarrow 5000 \left\{ \cos 30^\circ - \left(1 - \frac{1}{2} \right) \times \frac{1}{2} \right\} = \frac{1}{2} \times 500 \times v^2$$

$$\Rightarrow 10 \left\{ \frac{\sqrt{3}}{2} - \frac{1}{4} \right\} = \frac{1}{2} \times v^2$$

$$\Rightarrow v^2 = 10 \left\{ \sqrt{3} - 0.5 \right\}$$

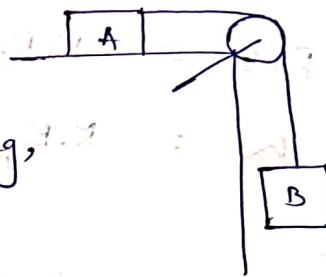
$$\Rightarrow v = \sqrt{10(\sqrt{3} - 0.5)} = 3.51 \text{ m/s}$$

Q. 2 blocks are joint by an inextensible cable as shown in figure.

If the syst. is released

from rest, find v_A after it has travelled a dist. of 2 meter. ($\mu = 0.25$). $M_A = 200 \text{ kg}$,

$$M_B = 300 \text{ kg}.$$



Solⁿ:

$$T - \frac{490}{5} = 200a \quad \text{--- (i)}$$

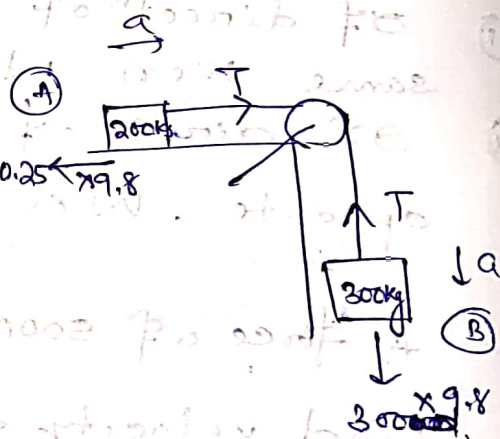
$$2940 - T = 300a \quad \text{--- (ii)}$$

$$2450 = 500a$$

$$\Rightarrow a = \frac{245}{5 \times 10} = 4.9 \text{ m/s}^2$$

$$v_A^2 = 2 \times 4.9 \times 2$$

$$v_A = \sqrt{4 \times 4.9} = 4.42 \text{ m/s}$$



11.10. (By work energy principle do it) :-
 (Assign marks) A block of mass 5 kg. resting on a 30° inclined plane is released. The block after travelling a distance of 0.5 m. along the inclined plane hits a spring of 15 N/cm . Find the compression on the spring. ($\mu = 0.2$, $m = 5 \text{ kg}$).

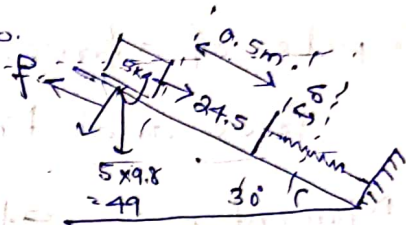
Soln:-

$$\left(24.5 - 0.2 \times 49 \times \frac{\sqrt{3}}{2} \right) \times 0.5$$

$$= \frac{1}{2} \times 5 \times v^2$$

KE₂

$$v_1 = 0$$



$$\Rightarrow KE_2 = 8 \text{ J}$$

$$\frac{1}{2} kx^2 = 8$$

$$\Rightarrow 15 \times 100 \times x^2 = 16$$

$$\Rightarrow x = \sqrt{\frac{16}{15 \times 100}} = \frac{4}{\sqrt{15 \times 10}} = 0.1 \text{ m.}$$

(3) Impulse - Momentum theorem.

$$\text{Impulse} = \text{Impulsive force} \times \Delta t$$

- ① Impulsive force: It is defined as the large amount of force which is developed in a small interval of time during collision.
- ② Impulse of a force: It is the product of impulsive force and time interval.
- ③ Momentum: It is the product of mass and velocity.
- ④ Impulse - Momentum theorem:

$$F = ma \quad \text{--- 2nd law}$$

$$F = m \frac{dv}{dt}$$

$$\Rightarrow \int_{t_1}^{t_2} F dt = \int_{v_1}^{v_2} m dv$$

$$\Rightarrow F [t]_{t_1}^{t_2} = m [v]_{v_1}^{v_2}$$

$$\Rightarrow F (t_2 - t_1) = m (v_2 - v_1)$$

$$\Rightarrow F \Delta t = m v_2 - m v_1$$

$$\Rightarrow I = M_{\text{final}} - M_{\text{initial}}$$

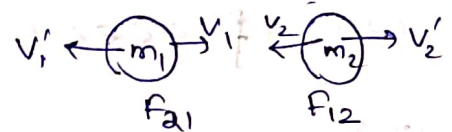
$$\Rightarrow I = \text{change in momentum.}$$

⑤ Law of conservation of momentum:

$I = \text{change in momentum.}$

Here m_1 and m_2 are masses of body 1 and 2 respectively,

v_1, v_2 are the velocity of m_1, m_2 before collision,
 v_1', v_2' are the velocity of " " after collision.



$$I = F_{12} \times \Delta t = F_{21} \times \Delta t$$

$$\Rightarrow \Sigma I = 0$$

$\Rightarrow \text{Change in momentum} = 0$

$\Rightarrow \text{Final momentum} - \text{Initial momentum} = 0$

$\Rightarrow \text{Initial momentum} = \text{Final momentum}$

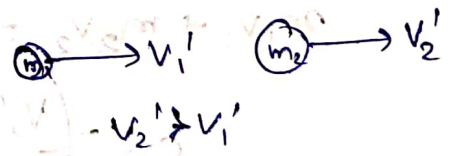
$$\Rightarrow (m_1 v_1 + m_2 v_2) = (m_1 v_1' + m_2 v_2')$$

⑥ Coefficient of restitution (e):-



$$v_1 > v_2$$

$v_1 - v_2 \rightarrow \text{velocity of approach}$



$v_2' - v_1' \rightarrow \text{velocity of separation.}$

$$e = \frac{\text{velocity of separation}}{\text{velocity of approach}} = \frac{v_2' - v_1'}{v_1 - v_2}$$

$$0 \leq e \leq 1$$

⑦ Types of Impulse :

- (i) If $e = 1$; then the impact is called perfectly elastic impact.
- (ii) If $0 < e < 1$; impact is called semi elastic impact.
- (iii) If $e = 0$; impact is called perfectly plastic impact.

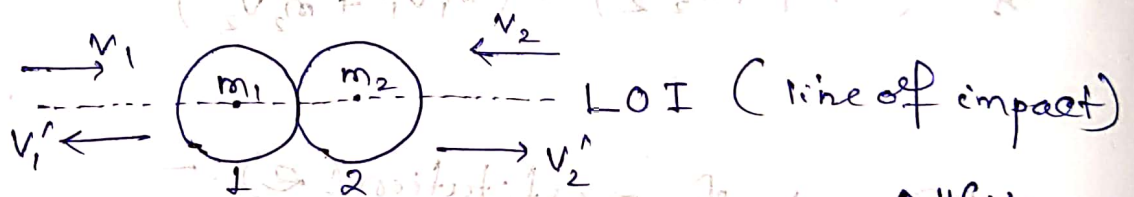
for $e = 0$, $v_2' - v_1' = 0$

$$\Rightarrow v_2' = v_1' = v'$$

$$m_1 v_1 + m_2 v_2 = m_1 v' + m_2 v' = (m_1 + m_2) v'$$

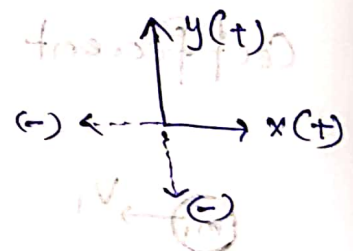
⑧ Types of impact (based on direction of velocities)

(i) Direct impact :-

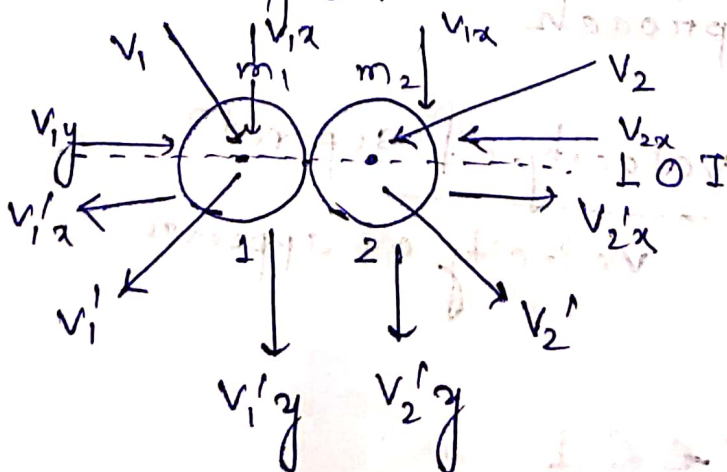


$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

$$e = - \frac{v_2' - v_1'}{v_2 - v_1}$$



(ii) Indirect impact :



$$v_{1y} = v_{1y}' \quad \& \quad v_{2y} = v_{2y}'$$

$$(v_{1x} m_1 + v_{2x} m_2) = (v_{1x}' m_1 + v_{2x}' m_2)$$

$$e = - \left(\frac{v_{2x}' - v_{1x}'}{v_{2x} - v_{1x}} \right)$$

(a) Collision between 2 bodies when mass of one body is infinite.

Applying law of conservation of energy

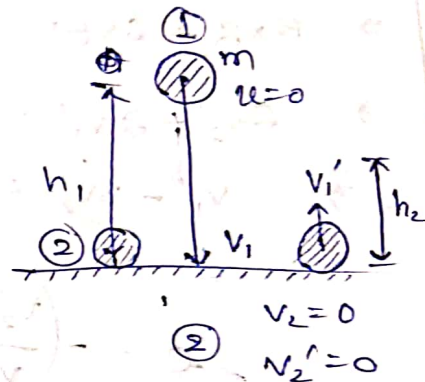
$$(K.E + P.E)_1 = (K.E + P.E)_2$$

$$\Rightarrow \frac{1}{2} m u^2 + mgh_1 = \frac{1}{2} m v_1^2 + 0$$

$$\Rightarrow mgh_1 = \frac{1}{2} m v_1^2$$

$$\Rightarrow \boxed{v_1 = \sqrt{2gh_1}}$$

Similarly $\boxed{v_1' = \sqrt{2gh_2}}$



$$e = (-) \frac{v_2' - v_1'}{v_2 - v_1} = \frac{-v_1'}{v_1}$$

$$\Rightarrow \boxed{v_1 \cdot e = -v_1'}$$

$$\Rightarrow \sqrt{2gh_1} \cdot e = -\sqrt{2gh_2}$$

$$\Rightarrow \boxed{e = -\frac{\sqrt{h_2}}{\sqrt{h_1}} = -\sqrt{\frac{h_2}{h_1}}}$$

~~a. find out.~~

Q. Two smooth discs collide with each other with initial velocity as shown in figure, find their velocity just after collision.

Soln:

Cons. of Momentum =

$$m_A V_1 + m_B V_2 = m_A V_1' + m_B V_2'$$

$$\Rightarrow 2 \times 2 + 6 \times (-5) = 2 V_1' + 6 V_2'$$

$$\Rightarrow 6 V_2' + 2 V_1' = 4 - 30$$

$$\Rightarrow V_1' - 3 V_2' = 13$$

$$e = - \left(\frac{V_2' - V_1'}{V_2 - V_1} \right)$$

$$\Rightarrow 0.8 = \frac{V_2' + V_1'}{-5 - 2}$$

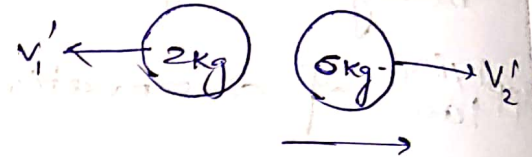
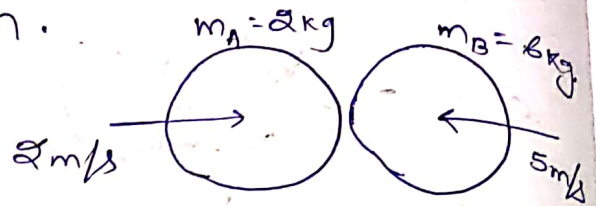
$$\Rightarrow V_1' + V_2' = -2.4 \quad \text{--- (II)}$$

$$\textcircled{I} + \textcircled{II}$$

$$-2 V_2' = 10.6$$

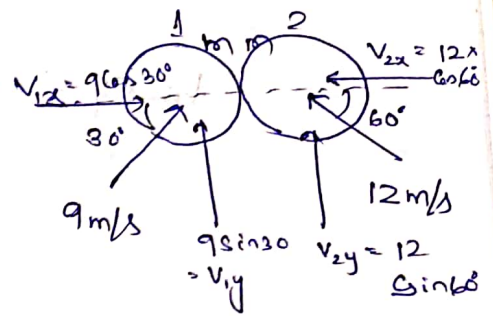
$$V_2' = -5.3 \text{ m/s}$$

$$V_1' = 5.3 - 2.4 = 2.9 \text{ m/s}$$



$$\begin{cases} V_1' = -7.4 \text{ m/s} \\ V_2' = -1.85 \text{ m/s} \end{cases}$$

Q. The magnitude and directions of two identical smooth ball before central oblique impact are as shown in figure. $e = 0.9$. Find the magnitude & direction of velocity of each ball after impact.



Soln:-

$$V_{1y}' = V_{1y} = 9 \sin 30^\circ$$

$$V_{2y}' = V_{2y} = 12 \sin 60^\circ$$

$$V_{1x} = 9 \cos 30^\circ = 7.79 \text{ m/s}$$

$$V_{1y} = 9 \sin 30^\circ = 4.5 \text{ m/s}$$

$$V_{2x} = -6 \text{ m/s}$$

$$V_{2y} = 12 \sin 60^\circ = 10.39 \text{ m/s}$$

$$e = - \left(\frac{V_{2x}' - V_{1x}'}{V_{2x} - V_{1x}} \right)$$

$$\Rightarrow 0.9 = - \left(\frac{V_{2x}' + V_{1x}'}{-6 - 7.79} \right)$$

$$\Rightarrow V_{1x}' + V_{2x}' = 12.41 \quad \text{--- (I)}$$

Conservation of Momentum:-

$$mV_{1x} + mV_{2x} = mV_{1x}' + mV_{2x}'$$

$$\Rightarrow 7.79 - 6 = V_{1x}' + V_{2x}'$$

$$\Rightarrow -V_{1x}' + V_{2x}' = 1.79$$

$$\Rightarrow V_{2x}' - V_{1x}' = 1.79$$

$$2V_{2x}' = 14.20$$

$$V_{2x}' = 7.10 \text{ m/s}$$

$$V_{1x}' = 5.31 \text{ m/s}$$

$$V_1' = \sqrt{V_{1x}'^2 + V_{1y}'^2}$$

$$= \cancel{12.5} 6.96 \text{ m/s}$$

$$V_2' = \sqrt{V_{2x}'^2 + V_{2y}'^2}$$

$$= 12.5 \text{ m/s}$$

$$\tan \theta_1 =$$

$$\tan \theta_2 =$$



$$\left(\frac{1 \text{ m/s}}{10 \text{ m/s}} + \frac{1 \text{ m/s}}{10 \text{ m/s}} \right) = 0.2$$

$$\left(\frac{1 \text{ m/s}}{10 \text{ m/s}} + \frac{1 \text{ m/s}}{10 \text{ m/s}} \right) = 0.2$$

$$12.5 = 1 \text{ m/s} + 1 \text{ m/s}$$

$$1 \text{ m/s} + 1 \text{ m/s} = 1 \text{ m/s} + 1 \text{ m/s}$$

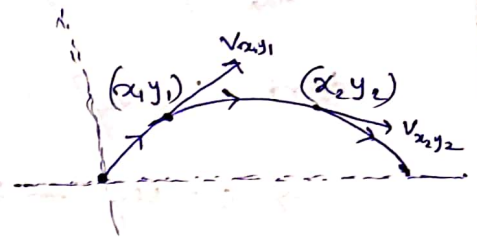
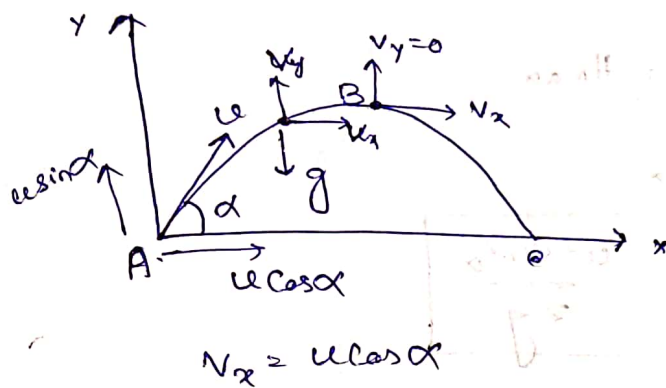
$$1 \text{ m/s} + 1 \text{ m/s} = 2 \text{ m/s}$$

$$1 \text{ m/s} + 1 \text{ m/s} = 2 \text{ m/s}$$

Module - 5

Curvilinear Translation :-

Projectile Motion:



Assumption:
air resistance = 0

x direction:

$$s = u_x t + \frac{1}{2} a_x t^2$$

$$s = u_x t$$

y direction:

$$v^2 = u^2 + 2as$$

$$v = u + at$$

* Time taken by the body to reach maxi. height:-

$$u_y = u \sin \alpha \quad v_y = 0 \quad a_y = -g$$

$$v_y = u_y + a_y t$$

$$\Rightarrow 0 = u \sin \alpha - g t$$

$$\Rightarrow t = \frac{u \sin \alpha}{g}$$

* Time of flight :-

$$T = 2t = \frac{2u \sin \alpha}{g}$$

* Maximum Height :-

$$H_{\max} = S_y = u_y t + \frac{1}{2} a_y t^2$$

$$\therefore V_y^2 = u_y^2 + 2a_y H_{\max}$$

$$\Rightarrow 0 = u_y^2 - 2g H_{\max}$$

$$\Rightarrow \boxed{H_{\max} = \frac{u_y^2}{2g} = \frac{u^2 \sin^2 \alpha}{2g}}$$

* Horizontal distance travelled or Range :-

$$R = S_x = u_x T = u \cos \alpha \cdot \frac{2u \sin \alpha}{g}$$

$$= \frac{u^2 \cdot 2 \sin \alpha \cdot \cos \alpha}{g} = \frac{u^2 \sin 2\alpha}{g}$$

$$\boxed{R = \frac{u^2 \sin 2\alpha}{g}}$$

* Maxi. Range :

For $\alpha = 45^\circ$, R is maximum. $= \frac{u^2}{g}$

* Velocity at given time (t) :-

At any time t ,

$$V_y = u \sin \alpha - gt, \quad V_x = u \cos \alpha$$

$$V = \sqrt{V_x^2 + V_y^2}$$

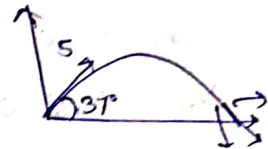
$$= \sqrt{u^2 \cos^2 \alpha + u^2 \sin^2 \alpha + g^2 t^2 - 2gu \sin \alpha t}$$

if θ is the angle between V_x & V_y ,

$$\tan \theta = \frac{V_y}{V_x}$$

Q. A ball is thrown with 5 m/s at angle of projection 37° . find T, H, R, express V_i & V_f in vector form.

Soln:- find velocity vector & speed of particle at 0.5 s.



$$T = \frac{2u \sin \theta}{g} = \frac{2 \times 5 \times \frac{3}{5}}{9.8}$$

$$= \frac{6}{9.8} = 0.61 \text{ s.}$$

$$H = \frac{u^2 \sin^2 \theta}{2g} = \frac{5^2 \times \frac{9}{25}}{2 \times 9.8} = \frac{9}{2 \times 9.8} = 0.45 \text{ m.}$$

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{5^2 \times 2 \times \frac{3}{5} \times \frac{4}{5}}{9.8} = \frac{24}{9.8} = 2.44 \text{ m.}$$

$$V_i = 5 \times \frac{4}{5} \hat{i} + 5 \times \frac{3}{5} \hat{j} = 4\hat{i} + 3\hat{j}$$

$$V_f = 4\hat{i} - 3\hat{j}$$

$$V_{yt} = 5 \times \frac{3}{5} - 9.8 \times \frac{1}{2} = 3 - 4.9 = -1.9 \text{ m/s}$$

$$V_{xt} = 5 \times \frac{4}{5} = 4 \text{ m/s}$$

$$\vec{V}_t = 4\hat{i} - 1.9\hat{j}$$

$$|\vec{V}_t| = \sqrt{4^2 + (1.9)^2} = 4.42 \text{ m/s.}$$

Equation of Trajectory:

At t sec, velocity is $v = v_x + v_y$

x direction:

$$u_x = u \cos \alpha$$

$$a_x = 0$$

$$v_x = u \cos \alpha$$

$$s = u_x t + \frac{1}{2} a_x t^2$$

$$\Rightarrow x = u \cos \alpha \cdot t \quad \Rightarrow t = \frac{x}{u \cos \alpha}$$

y direction:

$$u_y = u \sin \alpha$$

$$a_y = -g, \quad s_y = u_y t + \frac{1}{2} a_y t^2$$

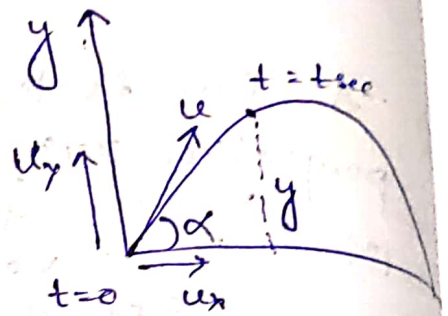
$$\Rightarrow y = u \sin \alpha t - \frac{1}{2} g t^2$$

$$\Rightarrow \boxed{y = u \sin \alpha t - \frac{1}{2} g \frac{x^2}{u^2 \cos^2 \alpha}}$$

$y \propto x^2$ so path is parabolic.

$$y = u \sin \alpha \cdot \frac{x}{u \cos \alpha} - \frac{1}{2} g \frac{x^2}{u^2 \cos^2 \alpha}$$

$$\boxed{y = x \tan \alpha - \frac{g}{2} \frac{x^2}{u^2 \cos^2 \alpha}}$$



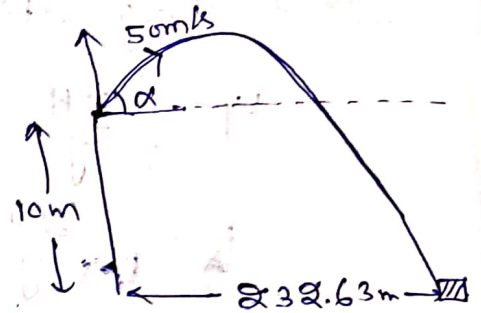
Q. A shell is fired at a velocity of 50m/s and it hits the target 232.63m away from gun, and 10m. below the level of it.

Soln:-

$$y = 50 \sin \alpha \cdot t - \frac{1}{2} g t^2$$

$$y = -10, u = 50 \text{ m/s}$$

$$x = 232.63 \text{ m.}$$



$$y = x \tan \alpha - \frac{g}{2u^2 \cos^2 \alpha} x^2$$

$$\Rightarrow -10 = 232.63 \cdot \tan \alpha - \frac{9.8}{2} \times \frac{(232.63)^2}{(50)^2 \times \cos^2 \alpha}$$

$$\Rightarrow -10 = 232.63 \tan \alpha - 4.9 \times \frac{(232.63)^2}{(50)^2} (1 + \tan^2 \alpha)$$

$$\Rightarrow 10 = 4.9 \times \frac{(232.63)^2}{(50)^2} \tan^2 \alpha - 232.63 \tan \alpha$$

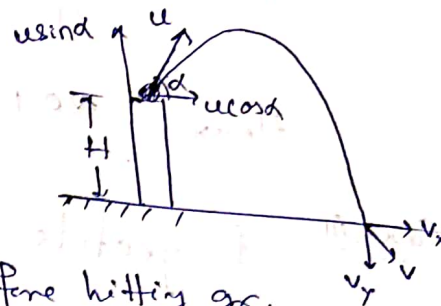
$$\Rightarrow 4.9 \times \frac{(232.63)^2}{50} \tan^2 \alpha - 232.63 \tan \alpha$$

$$+ 4.9 \times \frac{(232.63)^2}{(50)^2} - 10 = 0$$

Case-II

(Projectile at an angle)

Q. $u = 20 \text{ m/s}$, $\alpha = 30^\circ$
 $H = 20 \text{ m}$.



(i) find time taken to reach ground.

(ii) R (iii) velocity just before hitting ground.

(iv) time to reach max. height,
 (v) H_{max}

Soln: -

(i) $-20 = 20 \times \frac{1}{2} \times t - \frac{1}{2} \times 9.8 \cdot t^2$

$\Rightarrow -20 = 10t - 4.9t^2$

$\Rightarrow 4.9t^2 - 10t - 20 = 0$

$\Rightarrow t = 4 \text{ s. (i)}$

(ii) $R = 20 \times \frac{\sqrt{3}}{2} \times 4 = 60\sqrt{3} = 103.92 \text{ m}$.

(iii) $V_x = 20 \times \frac{\sqrt{3}}{2} = 10\sqrt{3} \text{ m/s}$.

$-v_y^2 = u^2 \sin^2 \alpha - 2g \cdot H$

$\Rightarrow v_y^2 = 2 \times 9.8 \times 20 - (20)^2 \times \frac{1}{4} = 167$

$\Rightarrow v_y = \sqrt{167} = 12.92 \text{ m/s}$

$V = \sqrt{v_x^2 + v_y^2} = 29.01 \text{ m/s}$

(iv) $0 = 20 \times \frac{1}{2} - 9.8 \times t$
 $10 = 9.8t$
 $t = 1.02 \text{ s}$

$$(iv) H = \frac{u^2 \sin^2 \theta}{2g} = \frac{(30)^2 \times \frac{1}{4}}{2 \times 9.8} = 11.47$$

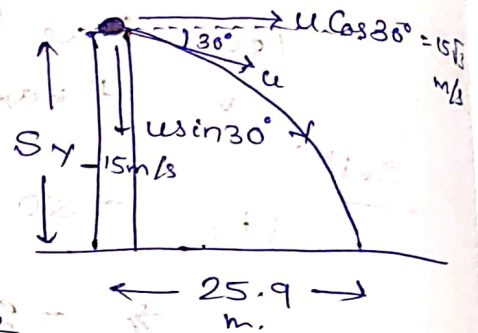
$$H_{\max} = 20 + 11.47 = 31.47 \text{ m. (from ground)}$$

Case-III Projectile at an angle (Downward)

$$u = 30 \text{ m/s}$$

$$R = 25.9 \text{ m}, S_y = ?, T = ?$$

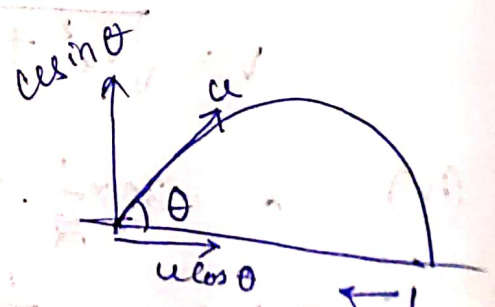
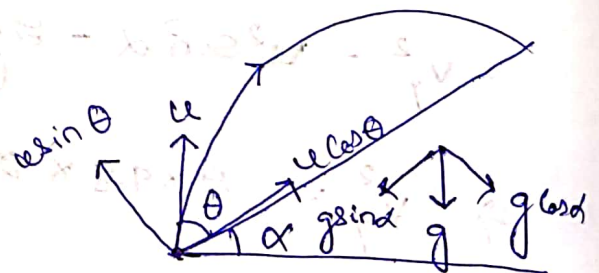
$$T = \frac{25.9}{15\sqrt{3}} = 0.99 \text{ s.} \approx 1 \text{ s.}$$



$$S_y = 15 \times 1 + \frac{1}{2} \times 9.8 \times 1^2 = 15 + 4.9 = 19.9 \text{ m.}$$

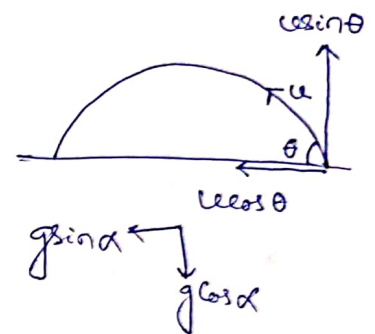
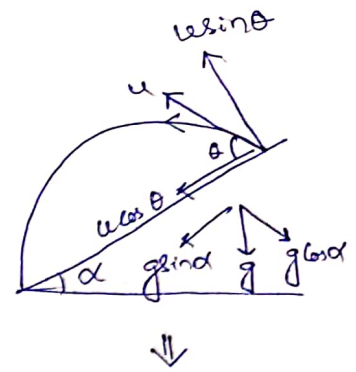
Projectile motion on inclined plane

Case-1 (Down to top)



Case-2

(Top to down)



$36^\circ = 15\sqrt{3}$
 m/s

$g \cos \alpha$

Circular Motion

Acceleration:-

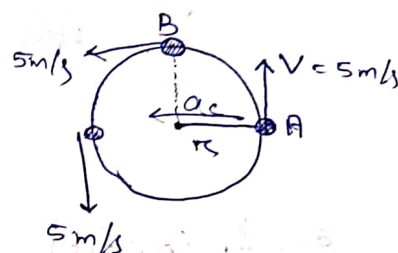
- 4 types - (i) Centripetal (a_c/a_r)
(ii) Tangential (a_t)
(iii) Net acceleration (a_{net})
(iv) Angular acceleration (α)

Centripetal Acceleration (a_c)

* It is due to change in direction of the velocity.

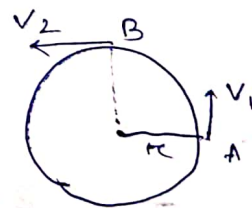
$$a_c = \frac{v^2}{r} = \frac{r^2 \omega^2}{r} = r\omega^2$$

Its direction is towards the center along the radius.



Tangential (a_t)

* It is due to change in magnitude of linear velocity and it acts tangential to the curve.

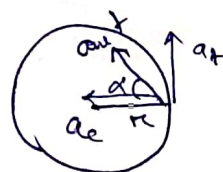


$a_t = \frac{dv}{dt}$, direction - tangential to the curve

Net Acceleration (a_{net})

* It is the resultant of $\vec{a_c}$ & $\vec{a_t}$

$$a_{net} = \left| \frac{dv}{dt} \right|, \quad a_{net} = \sqrt{a_t^2 + a_c^2}$$
$$\tan \alpha = \frac{a_t}{a_c}$$



Q. If $v = at$ & $r = 9m$, then find a_c at $t = 3 \text{ sec}$.

Soln:-

$$a_c = \frac{4t^2}{9} \Big|_{t=3} = 4 \text{ m/s}^2$$

$$a_t = \frac{d}{dt}(2t) = 2 \text{ m/s}^2$$

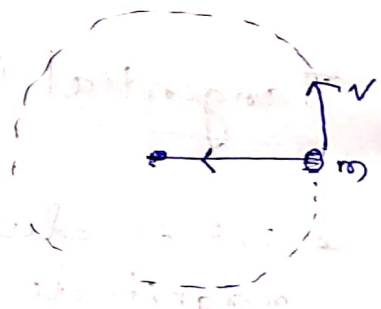
$$a_{\text{net}} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5} \text{ m/s}^2$$

Angular Acceleration (α) :-

$$\vec{\alpha} = \vec{r} \times \vec{\omega}, \quad \alpha = \frac{d\omega}{dt}$$

Centripetal force (f_c) :-

$$f_c = ma_c = \frac{mv^2}{r} = mr\omega^2$$



Q.

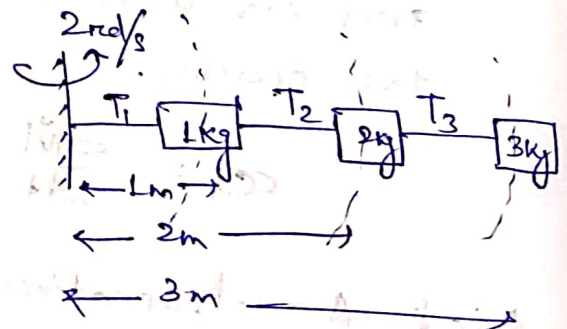
$$T_3 = 3 \times 3 \times 4 = 36 \text{ N}$$

$$T_2 - T_3 = 2 \times 2 \times 4 = 16$$

$$T_2 = 36 + 16 = 52 \text{ N}$$

$$T_1 - T_2 = 1 \times 1 \times 4 = 4$$

$$T_1 = 56 \text{ N}$$



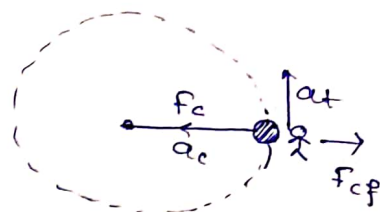
Centrifugal force:

* It is a pseudoforce.

(i) Inertial frame ($\vec{a} = 0$)

(ii) Non inertial frame ($\vec{a} \neq 0$)

$$F_{cf} = m\omega^2 r = \frac{mv^2}{r}$$



* Turning of a car on a Curve:

Condⁿ for not skidding,

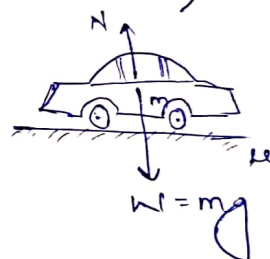
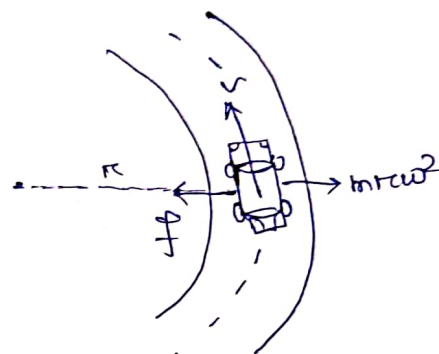
$$f \geq \frac{mv^2}{r}$$

$$\text{or } \mu N \geq \frac{mv^2}{r}$$

$$\text{or } \mu mg \geq \frac{mv^2}{r}$$

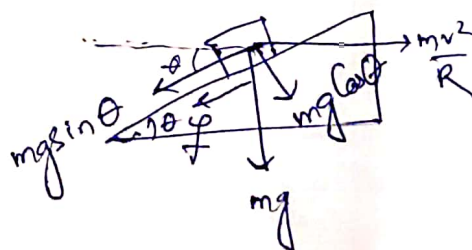
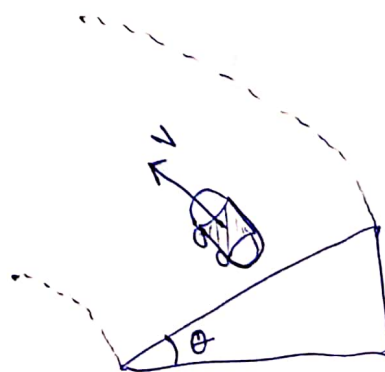
$$\boxed{\mu g \geq \frac{v^2}{r}}$$

$$\Rightarrow \boxed{v \leq \sqrt{\mu g r}}$$



* Banking of Road:

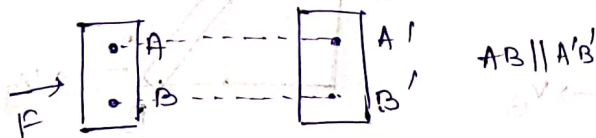
$$mg \sin \theta \cos \theta + f \cos \theta = \frac{mv^2}{R}$$



Rotational Motion (Rigid body motion)

* If deformation is zero; body is Rigid.
Also if the dist. between 2 point is same even if we apply force, body is said to be rigid body.

Translational Motion



(i) Rectilinear Translation.

(ii) Curvilinear Motion.



Rotational Motion

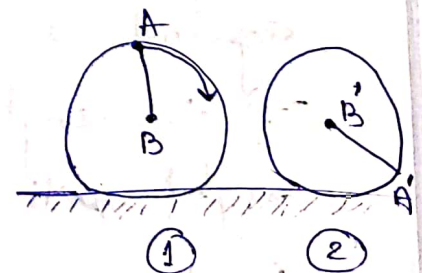


$AB \& A'B'$
are not
parallel.

one end (O)
must be fixed.

Planar Motion (Rolling)

→ It is the combination of translational and rotational motion.



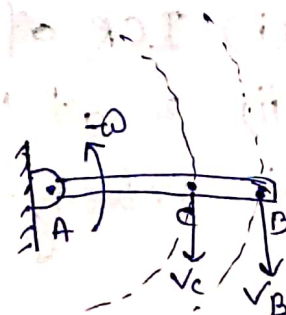
* Rotational Motion -

(i) Pure rotational:-

$$v = r\omega$$

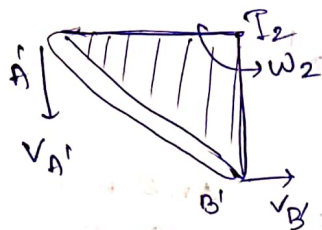
$$v_A = 0$$

Point 'A' is called center of rotation.



AB, AC : are called radii of rotation.
 v_B & v_C (the linear velocity) are $\perp^{\text{th}}$ to radii of rotation.

(2) General Plane Motion (ICR : Instantaneous Center of rotation.)

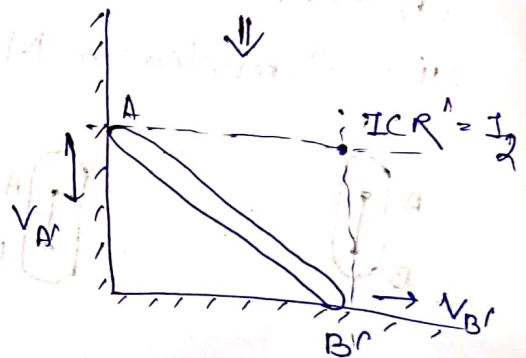
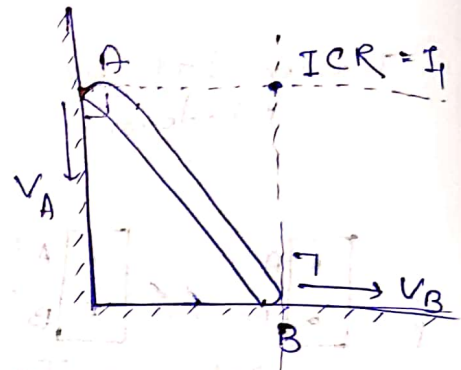
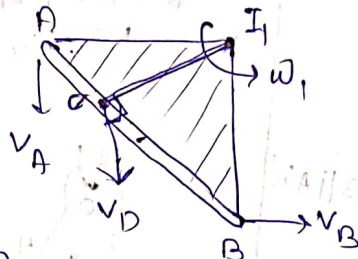


$$v_A = (AI_1) \omega_1$$

$$v_B = (BI_1) \omega_1$$

$$v_{A'} = (AI_2) \omega_2$$

$$v_{B'} = (BI_2) \omega_2$$

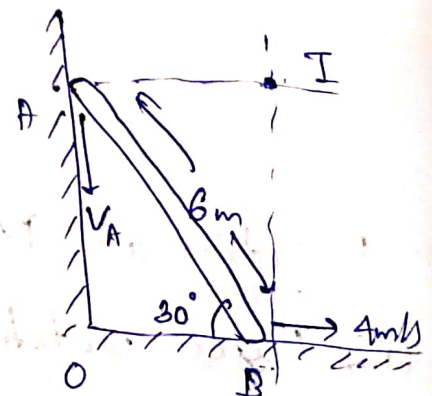


Q. Figure shows a ladder of 6m. Resting against a vertical wall A.

If the angle B of the foot towards right with a velocity 4m/s.

find

- (i) ICR of ladder
- (ii) ω at the instant
- (iii) velocity at end A



Solⁿ:

$$OB = 6 \times \frac{\sqrt{3}}{2} = 3\sqrt{3} \text{ m} = AI = 5.19 \text{ m}.$$

$$OA = 3 \text{ m} = BI$$

$$4\omega = 3 \times \omega$$

$$\Rightarrow \omega = \frac{4}{3} = 1.33 \text{ rad/s}.$$

$$V_A = r_{AI} \omega = 5.19 \times 1.33 = 6.9 \text{ m/s}.$$

Q. Block D moves with a speed of 3 m/s.
 V_B , ω_{BD} & ω_{AB} = ?

Solⁿ:

$$DI = BI = 0.4 \times \frac{1}{\sqrt{2}} = 0.28 \text{ m}.$$

$$3 = DI \times \omega_{BD}$$

$$\omega_{BD} = \frac{3}{0.28} = 10.71 \text{ rad/s}$$

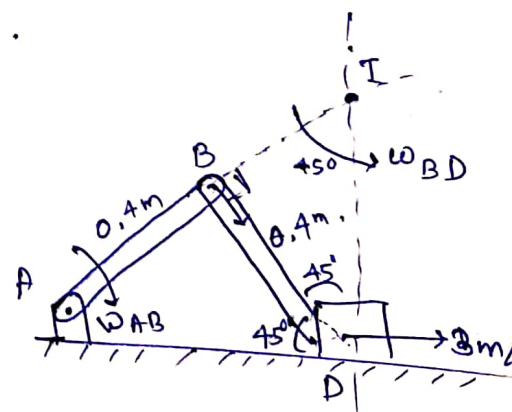
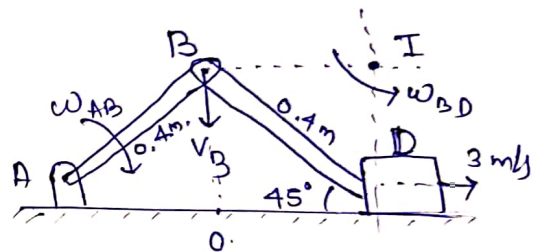
$$V_B = 0.28 \times 10.71 = 3 \text{ m/s}.$$

$$3 = DI \times \omega_{BD}$$

$$DI = 0.4 \times \frac{1}{\sqrt{2}} = 0.28$$

$$\omega_{BD} = \frac{3}{0.28} = 10.71 \text{ rad/s}.$$

$$V_B = BI \times 10.71 = \frac{0.4}{\sqrt{2}} \times 10.71$$



long Question

- (i) Truss
- (ii) Free body diagram
- (iii) Parallel force
- (iv) Coefficient of restitution & impulse-momentum, work-energy
- (v) Virtual work
- (vi) Moment of inertia & e.o.g.