B.Tech- 3rd

Mathematics-III

Full Marks : 50

Time : $2\frac{1}{2}$ hours

Answer **all** questions

The figures in the right-hand margin indicate marks

Symbols carry usual meaning

1. Answer *all* questions : 2×5

(*a*) Find the Laplace transform of $\frac{e^t}{\sqrt{t}}$.

(*b*) Use variable separation method to solve $u_{xy} = u_x$.

(*c*) Under what conditions a function can serve as the probability distribution of a discrete random variable ?

(*d*) Write is the probability function of uniform distribution. What is the mean of this distribution ?

(*e*) Define correlation coefficient r. What can you say about its range ?

2. (*a*) State second shifting theorem. Use this to find the inverse of $se^{-2s}/(s^2+\pi^2)$. 4

(*b*) Use Laplace transform to solve :

$$y''-4y'+3y=6t-8,\ y(0)=y'(0)=0.$$ 4

Or

(*a*) Using Laplace transform solve the following integral equation 4

$$y(t)=t+\int_0^t y(u)\sin(t-u)du.$$

(*b*) Use transform of derivatives to find the Laplace transform of t^n, $n\in I^+$. 4

(*Continued*)

3. (*a*) Find the Fourier series of the function

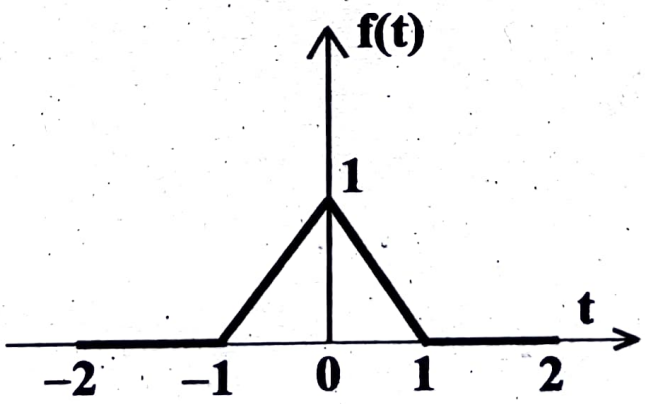
$$f(x)=\begin{cases}-k & \text{if } -\pi<x<0\\ k & \text{if } 0<x<\pi\end{cases}.$$

Also, show that $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+-\ldots=\frac{\pi}{4}$. 4

(*b*) Find the subsequent deflection $u(x,t)$ of a string of length $l=\pi$, when $c^2=1$, the initial velocity is zero, and the initial deflection is $0.01\sin 3x$. 4

Or

State whether the following function is even or odd, and find its Fourier series expansion. 8



4. (*a*) State and prove Baye's theorem. 4

(*b*) Construct a distribution function for the probability density function 5

$$f(x)=\begin{cases}\lambda x(1-x);0<x<1\\ 0;\ \text{otherwise}\end{cases}$$

Also, find $P\left(X>\frac{1}{2}\right)$. 4

Or

(*a*) If events A and B are two independent events, then show that

(*i*) A^c, B are independent events

(*ii*) A^c, B^c are independent events. 4

(*b*) A small filling station is supplied with gasoline every saturday afternoon. Assume that its volume X of sales in thousands of gallons has the probability density $f(x)=6x(1-x)$, $0<x<1$ and 0 otherwise.

Determine the mean, variance and standard variance. 4

5. (*a*) Show that mean and variance of Poisson distribution are equal. 4

(*b*) The length of a telephone conversion has an exponential distribution with a mean of 3 minutes. Find the probability that a call (i) ends in less than 3 minutes, (ii) takes between 3 and 5 minutes. 4

Or

(*a*) If the probability of hitting a target in a single target is 10% and 10 shots are fired independently, what is the probability that the target will be hit at least once ? 4

(*b*) If the lifetime X of a certain kind of automobile battery is normally distributed with a mean of 5 years and a standard deviation of 1 year, and the manufacture

wishes to guarantee the battery for 4 years. What percentage of the batteries needs to replace within the guarantee ? (Use $\phi(1) = 0.8413$) 4

6. (*a*) Apply maximum likelihood estimate for the parameter μ of the normal distribution with known variance $\sigma^2 = \sigma_0^2$. 4

(*b*) Find a 95% confidence interval for the mean μ of a normal population with standard deviation 1.2 using the sample 10, 10, 8, 12, 10, 11, 10, 11 and the value of $c = 1.96$. 4

Or

(*a*) Find the maximum likelihood estimation of Binomial distribution using p as parameter. 4

(*b*) Find the regression line of y on x for the data (-2, 3.5), (0, 1.5), (2,1), (4, -0.5), (6, -1). 4
